

Un peu de géométrie plane élémentaire !

Aziz EL KACIMI

Professeur émérite à

Université Polytechnique Hauts-de-France - Valenciennes

Mathématicien en résidence à

Cité des Géométries - Gare Numérique de Jeumont

Séances de formation

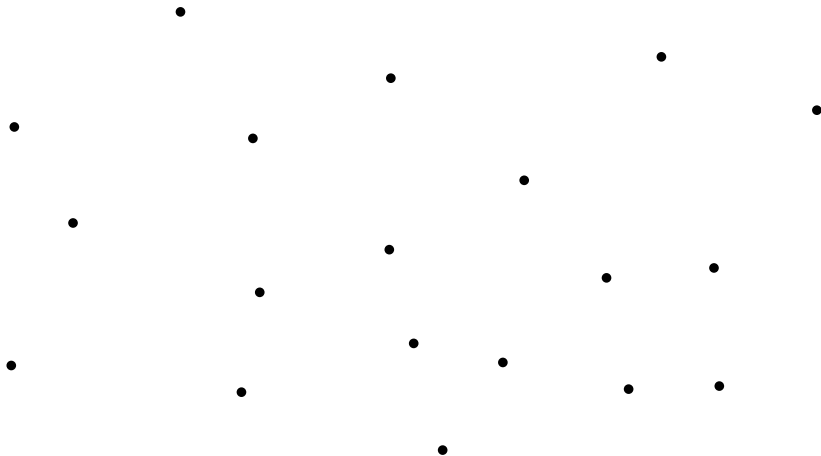
au

Musée de la Dentelle - Caudry

Les 6 et 13 août 2019

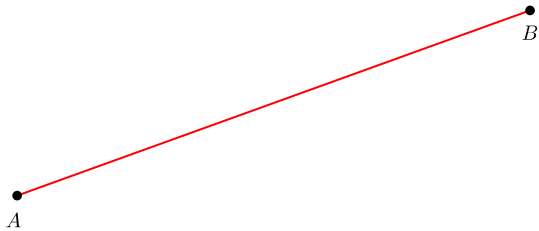
1. Objets et outils de base

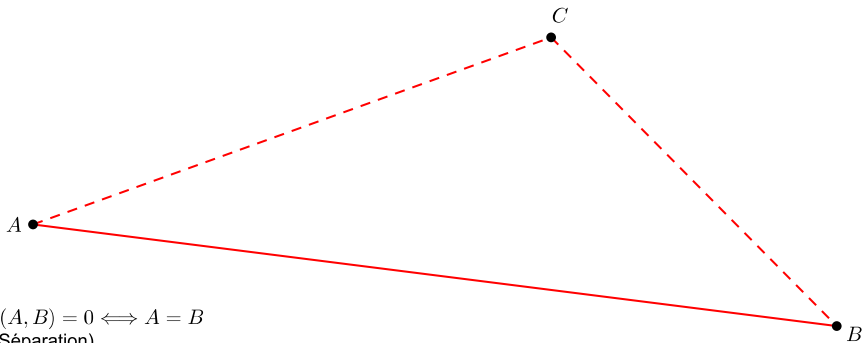
Le plan et ses éléments



•
A

•
B





$$d(A, B) = 0 \iff A = B$$

(Séparation)

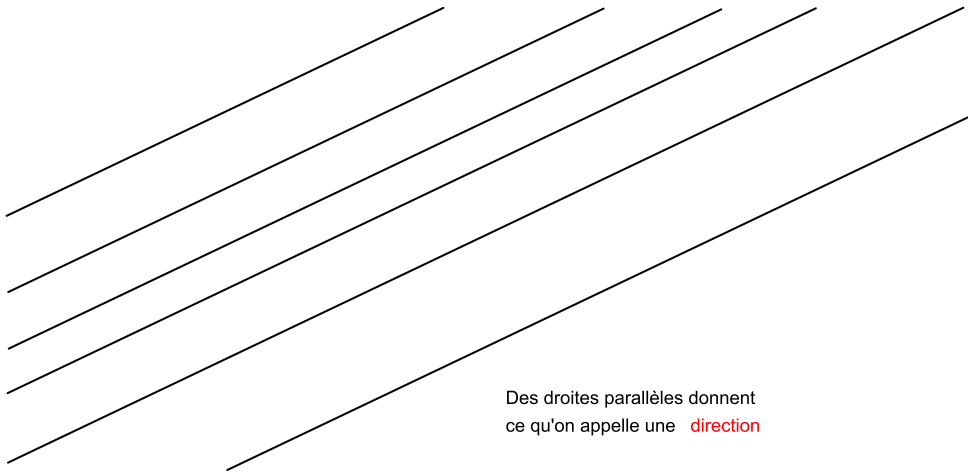
$$d(A, B) = d(B, A)$$

(Symétrie)

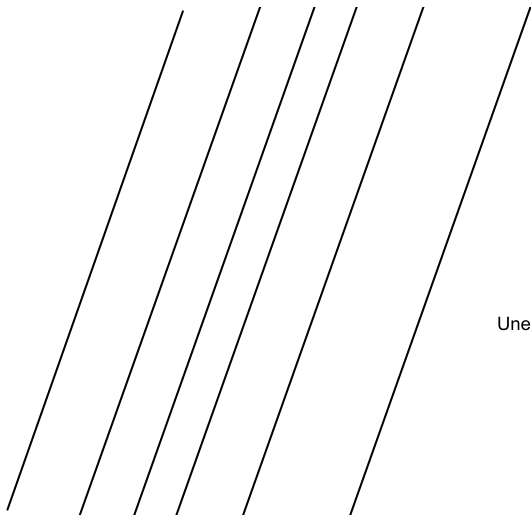
$$d(A, B) \leq d(A, C) + d(C, B)$$

(Inégalité du triangle)

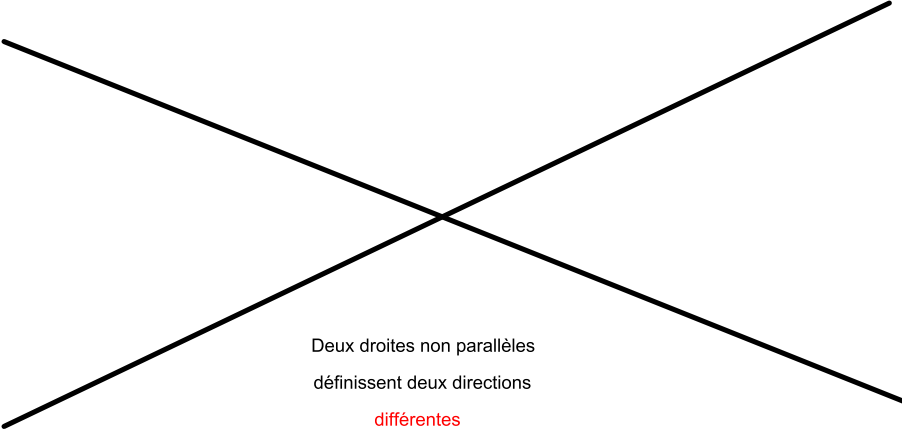
La notion de direction



Des droites parallèles donnent
ce qu'on appelle une **direction**

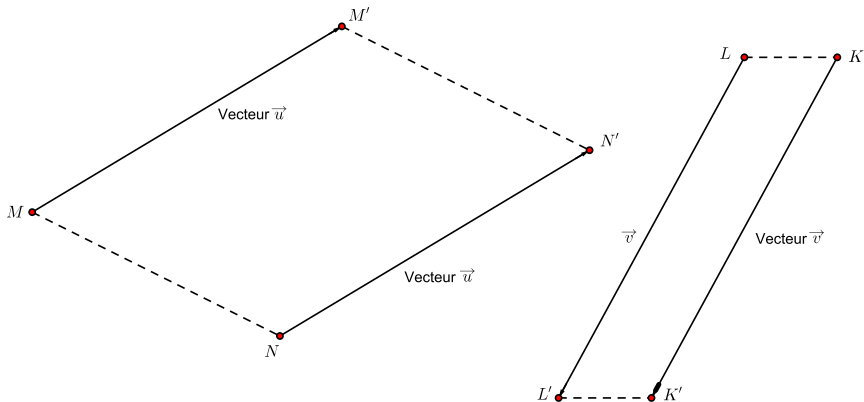


Une autre **direction**



Deux droites non parallèles
définissent deux directions
différentes

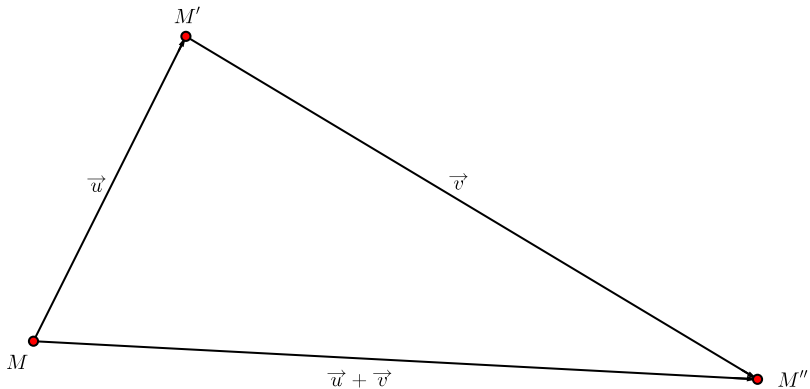
La notion de vecteur



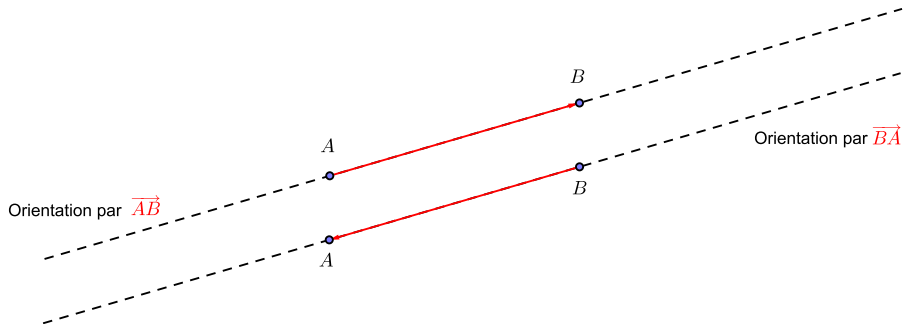
Un vecteur est donc défini par :

- sa **direction** : donnée par une droite qui lui est parallèle ;
- son **sens** ;
- sa **longueur**.

Somme de vecteurs



La notion d'orientation

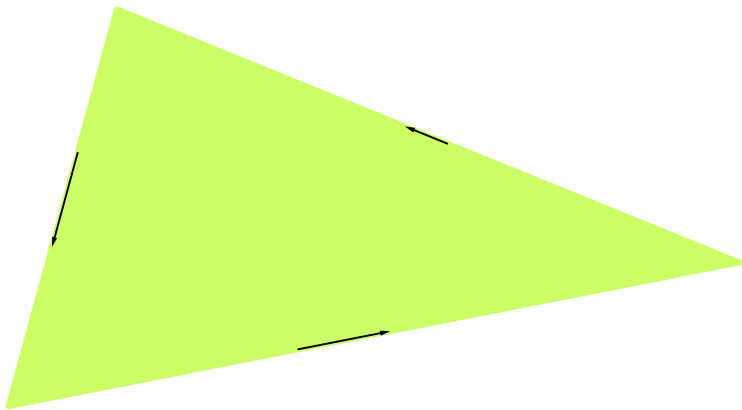


Une droite donnée définit une seule direction avec deux orientations

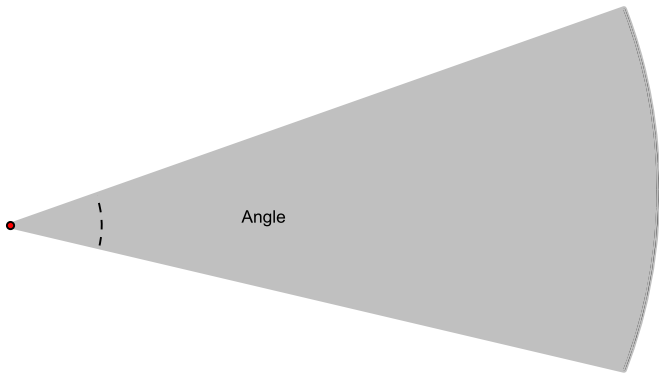
Mais comment orienter le plan ?

L'orientation est positive si l'étang est à gauche quand on se déplace sur son bord, ce qui est le cas.

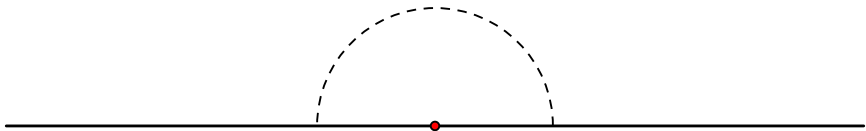
En particulier, voici un triangle orienté positivement !



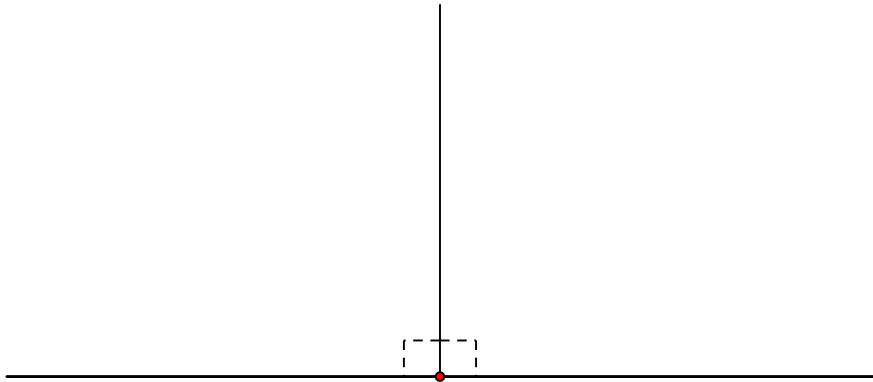
La notion d'angle



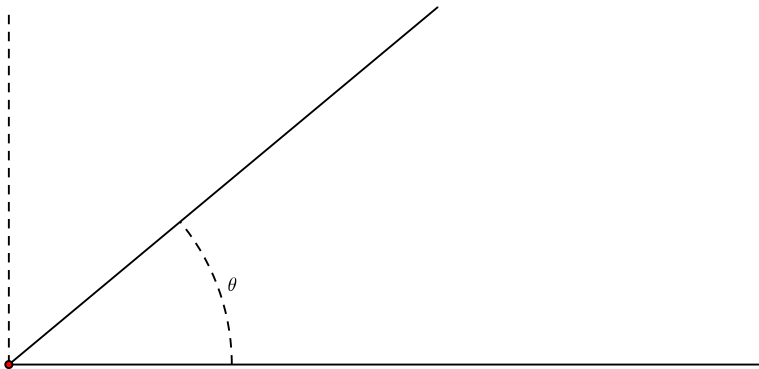
Angle plat



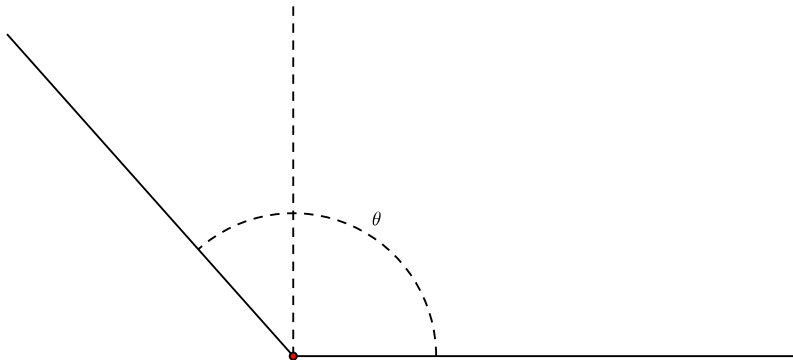
Angle droit



Angle aigu



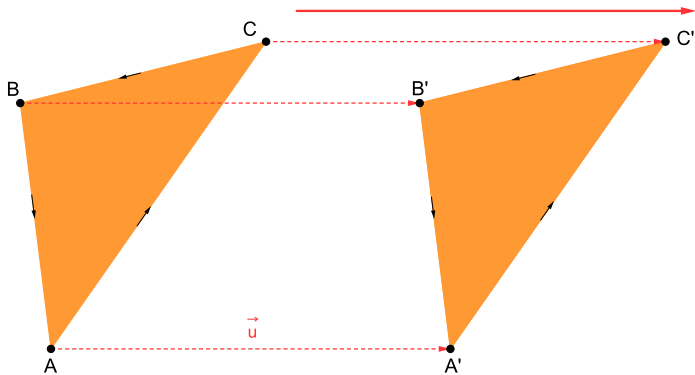
Angle obtu



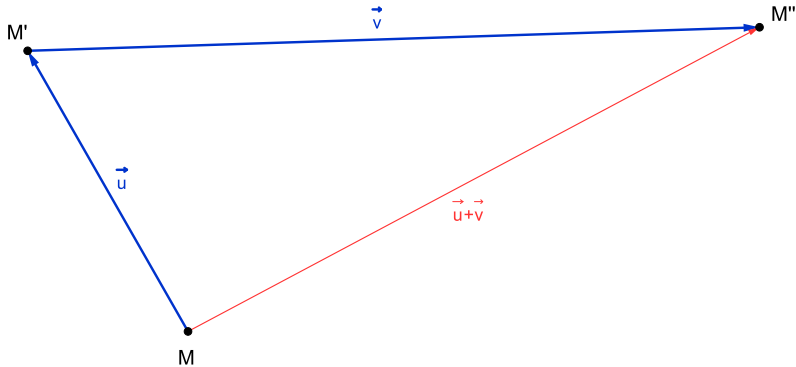
2. Mouvements euclidiens

Translation

Elle consiste à pousser les points par un vecteur \vec{u} . C'est un **mouvement** (ou transformation) qui **conserve l'orientation**.

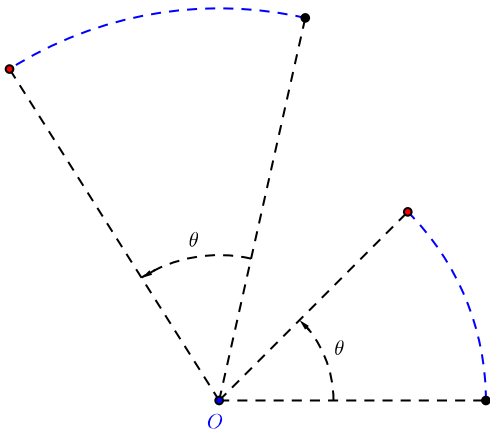


Les translations se composent comme les vecteurs.
La translation de vecteur nul est l'identité :
elle ne fait bouger aucun point.

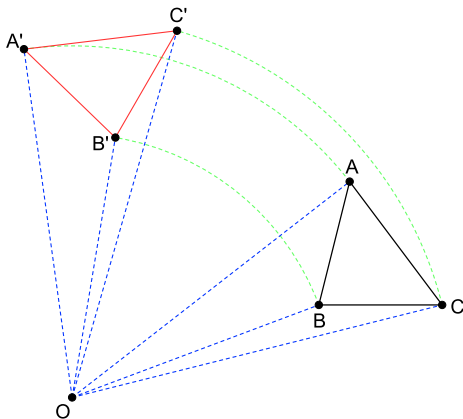


Rotation

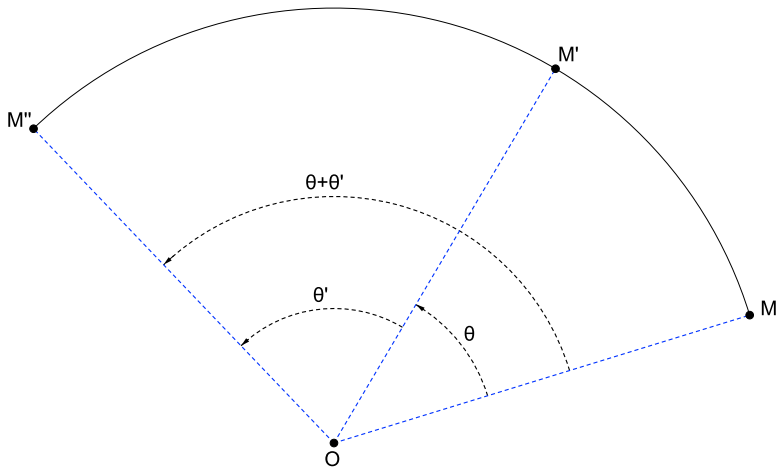
Elle consiste à faire tourner les points d'un angle θ appelé **angle de rotation** autour d'un point O appelé **centre de rotation**.

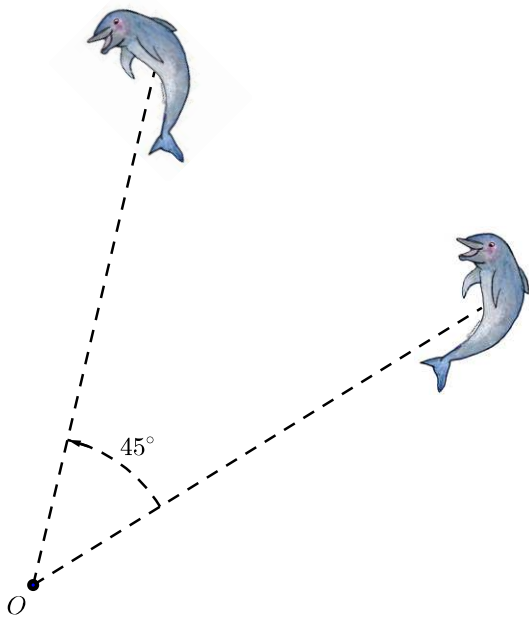


C'est un mouvement euclidien qui conserve l'orientation.



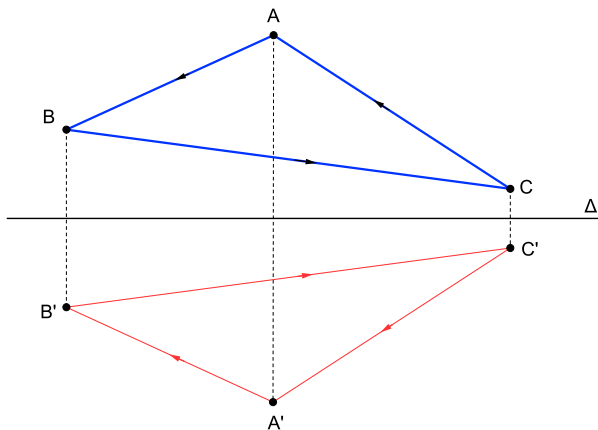
La composée de deux rotations de centre O et d'angles respectifs θ et θ' est une rotation de même centre O et d'angle $\theta + \theta'$.



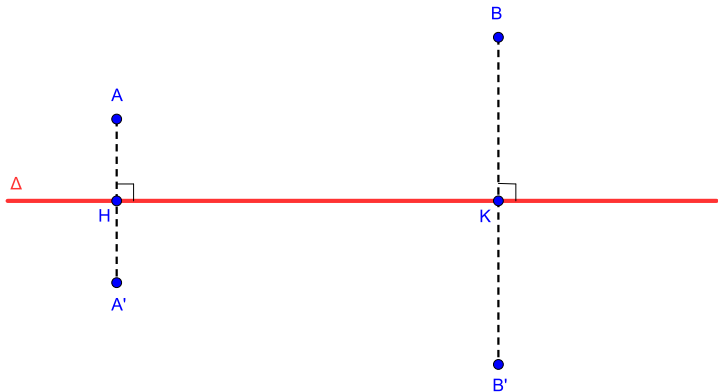


Réflexion ou symétrie axiale

C'est regarder l'image d'un objet dans un miroir ! Elle est donc associée à une droite appelée **axe de réflexion** (ou **axe de symétrie**). Elle ne conserve pas l'orientation.

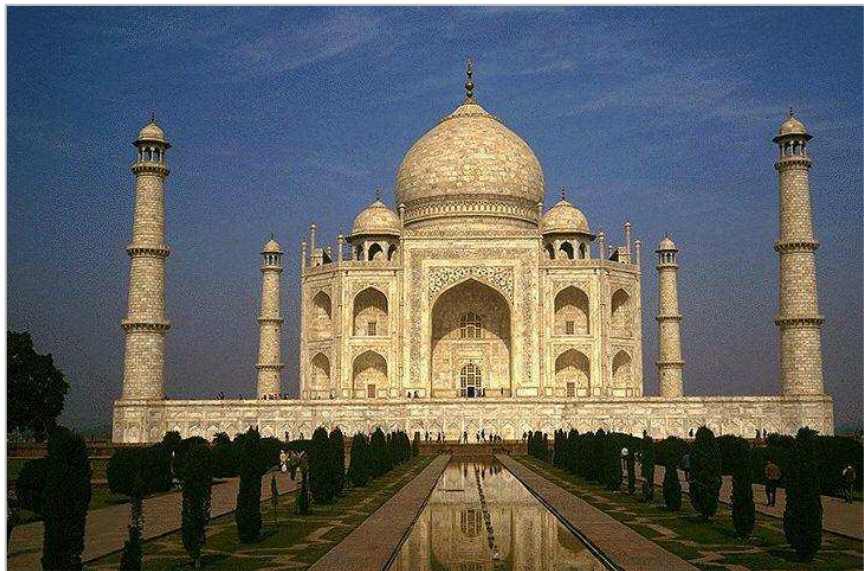


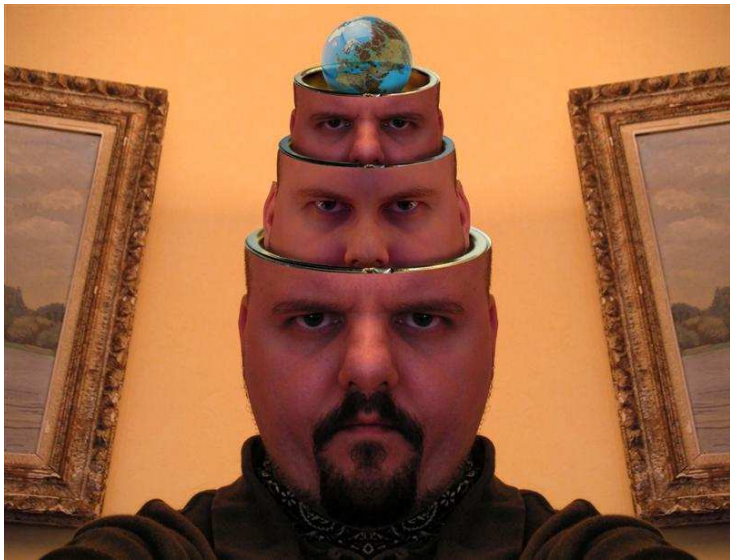
- Une **symétrie axiale** est définie par une droite Δ .
- À tout point A correspond un point A' tel que :
 - ① $A = A'$ si A est sur Δ , sinon
 - ② la droite (AA') est perpendiculaire à Δ en un point H et on a l'égalité des longueurs $AH = HA'$.
- On dira que Δ est l'**axe** de la symétrie.

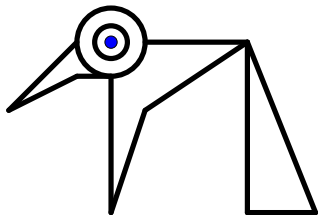
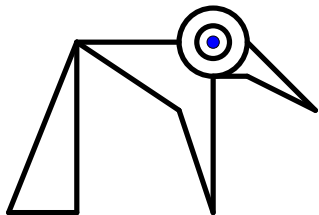


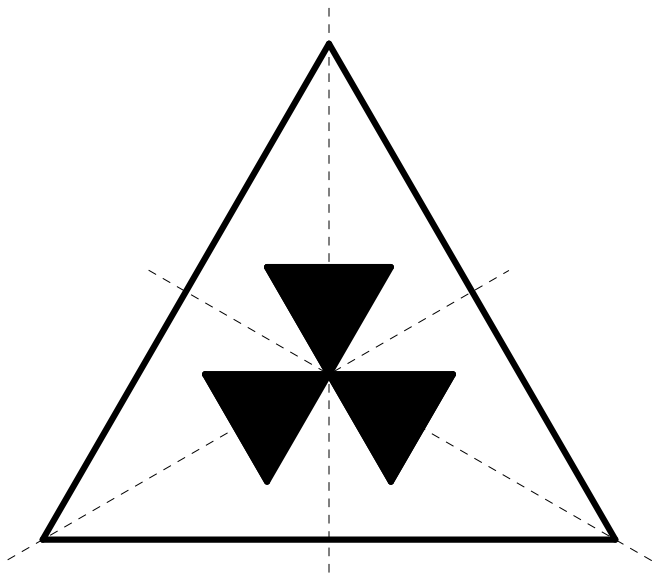




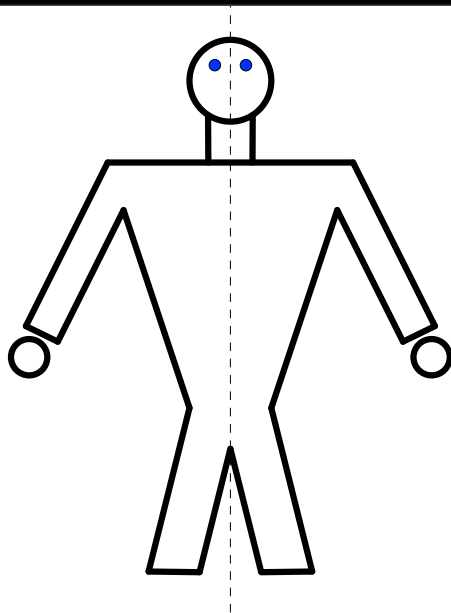




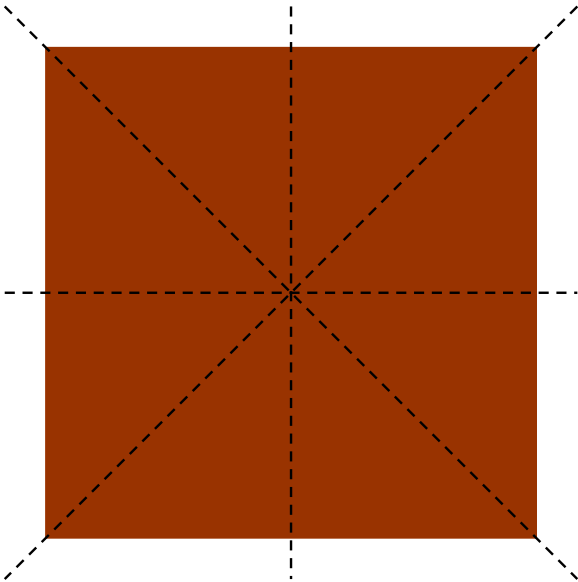


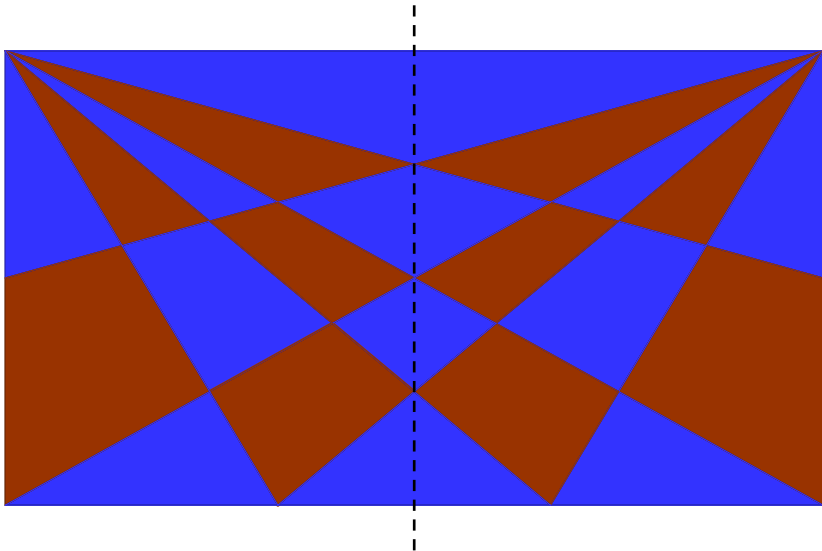


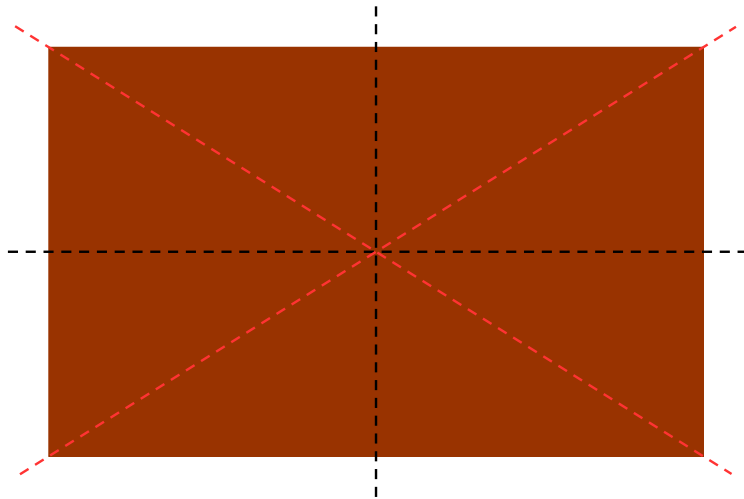
Attention : risque d'irradiation !

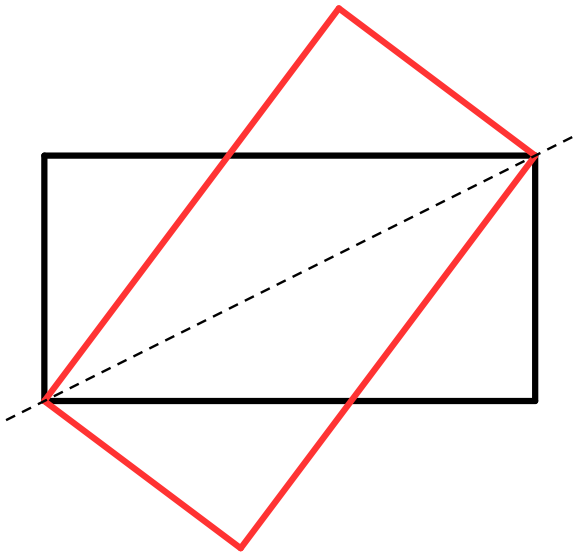


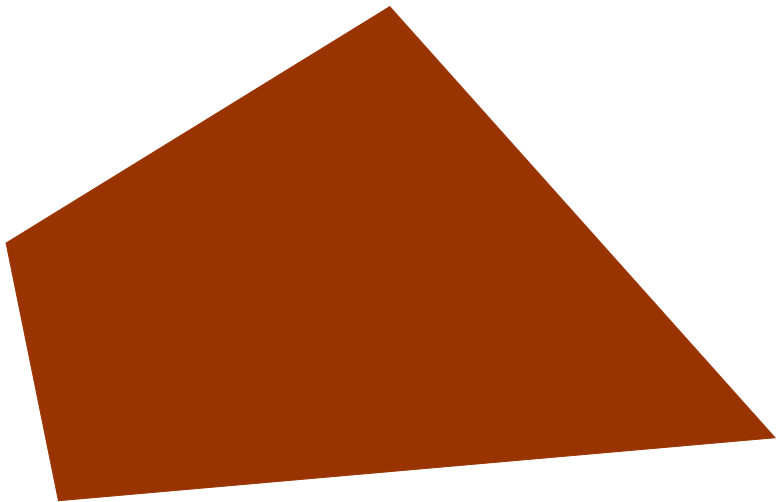
Premier étage à droite !

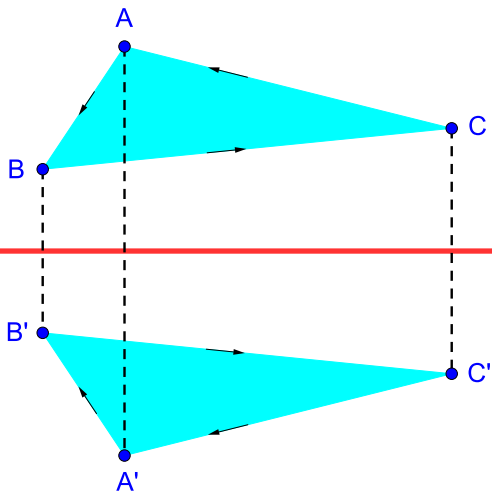


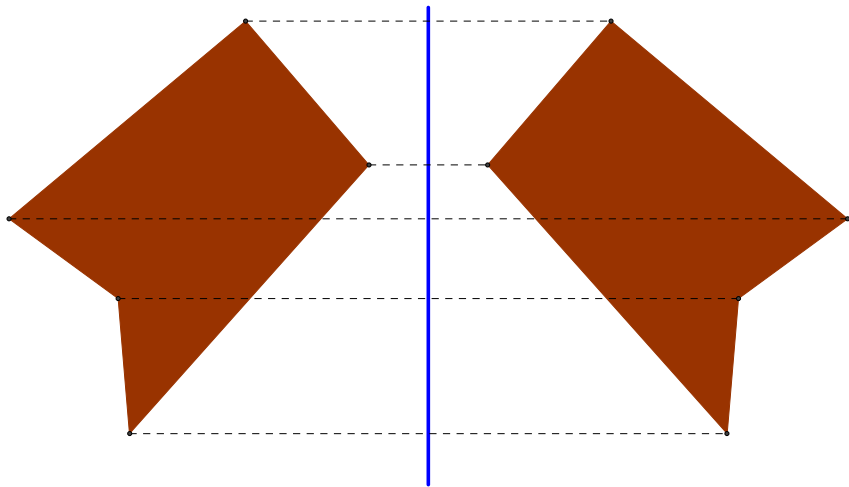


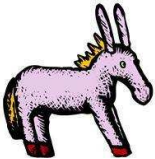




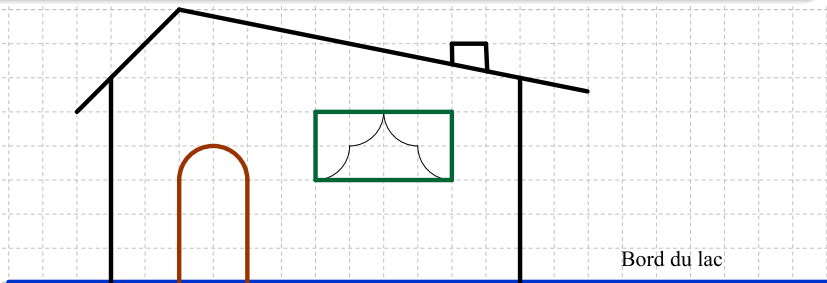




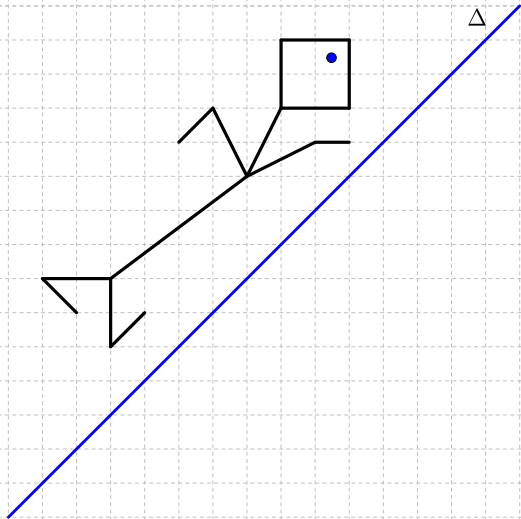




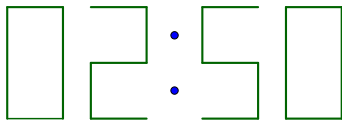
Cette maison est située au bord d'un lac.
Dessiner son reflet dans l'eau.



En utilisant uniquement une règle non graduée, tracer le symétrique du bonhomme par rapport à la droite.

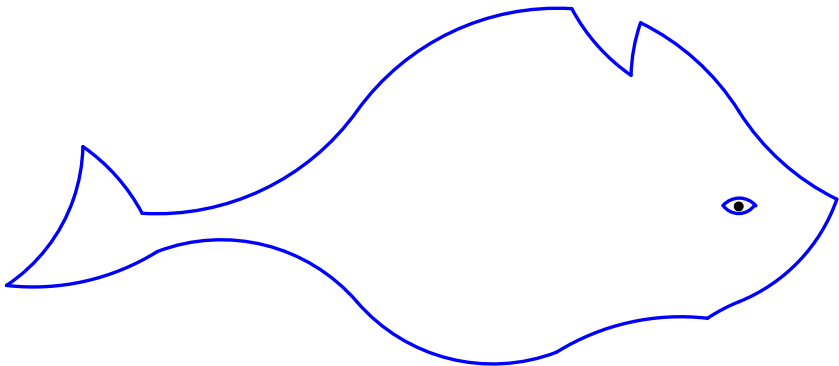


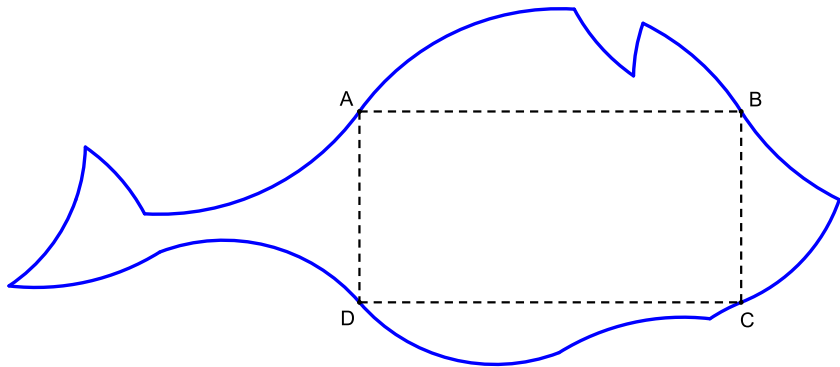
Donner tous les chiffres qui admettent un axe de symétrie (vertical, horizontal...), un centre de symétrie ainsi que toutes les heures symétriques.

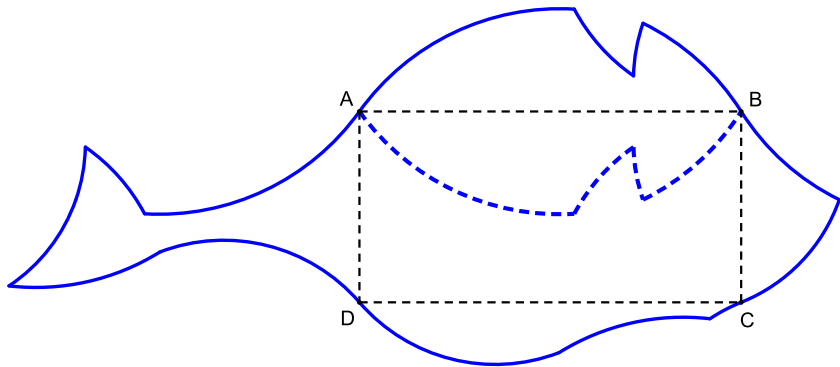


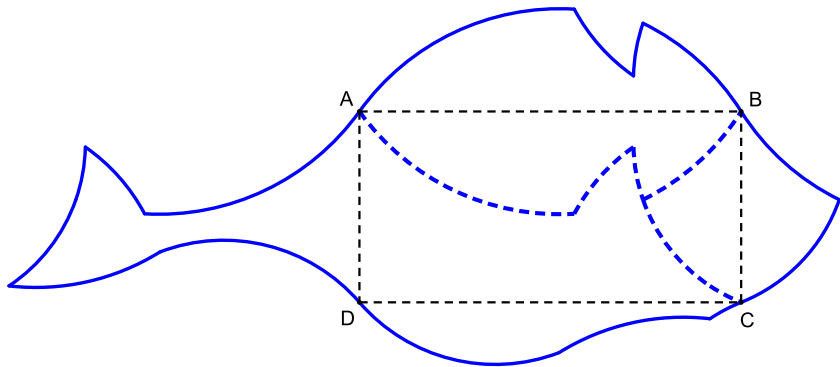
Heure symétrique !

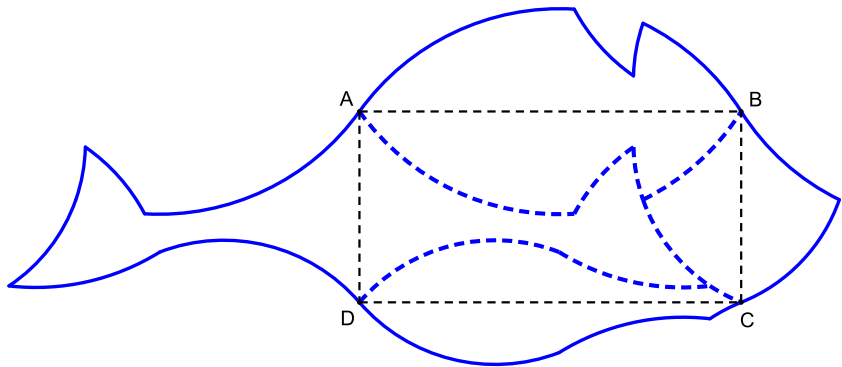
Mangez du poisson !



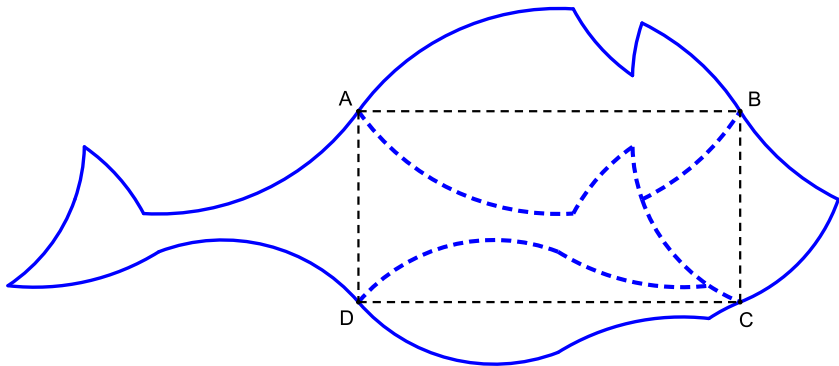


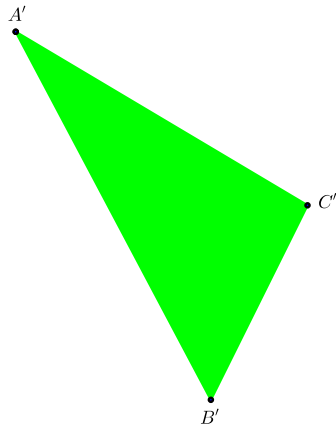
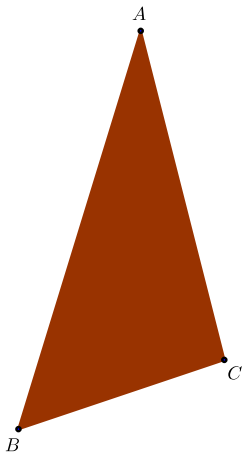






Le rectangle $ABCD$ a pour longueur 20cm et pour largeur 10cm .
Quelle est l'aire du poisson ?





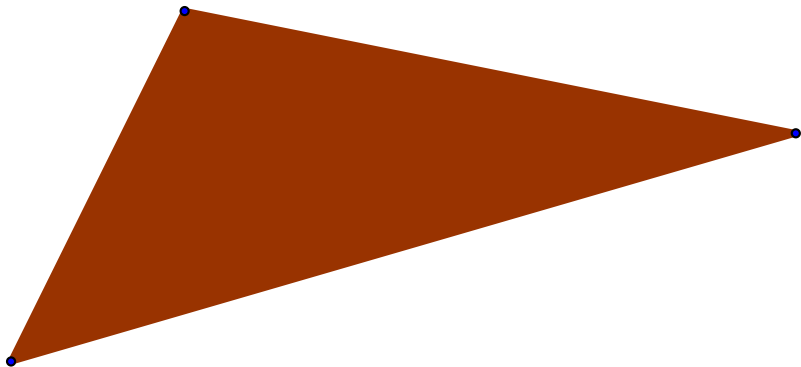


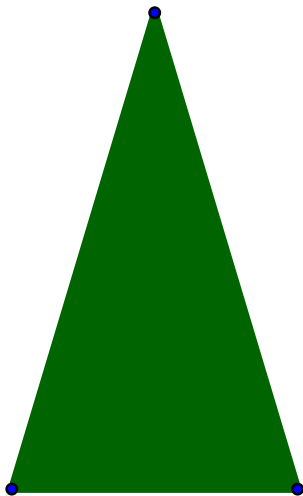




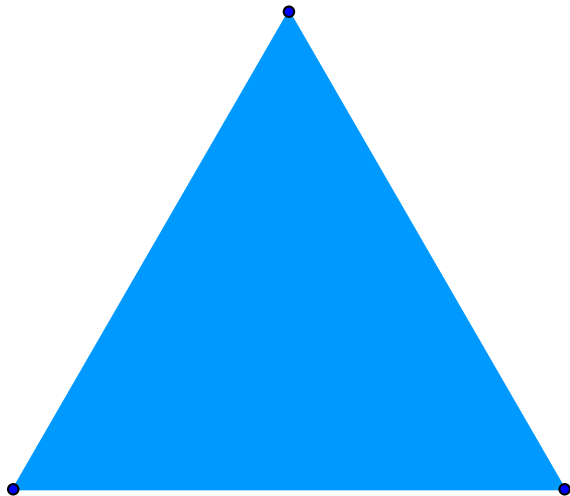
3. Figures géométriques

Des triangles

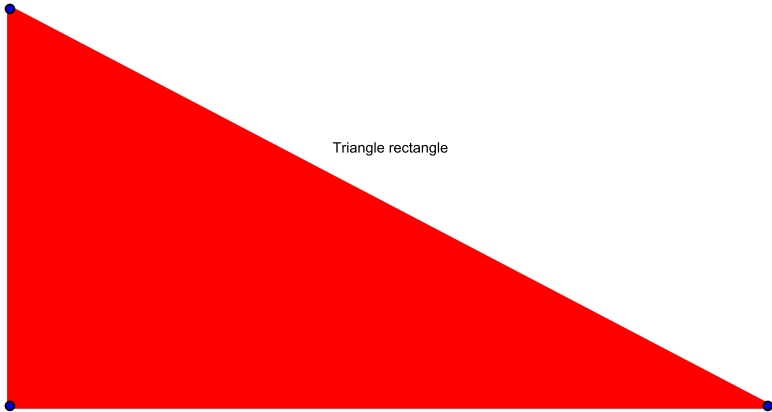




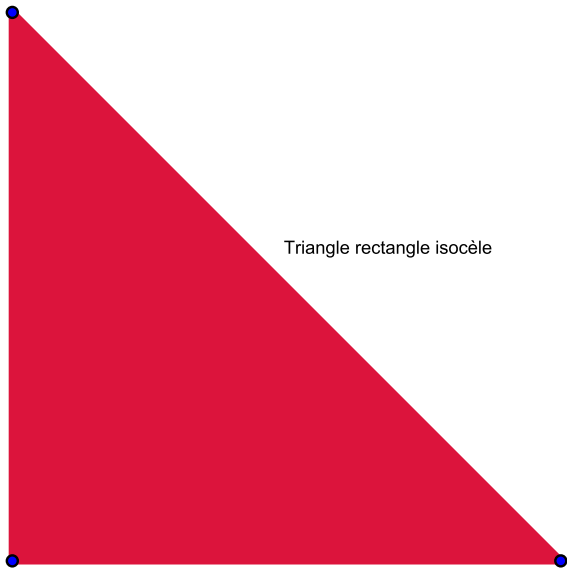
Triangle isocèle



Triangle équilatéral



Triangle rectangle



Triangle rectangle isocèle

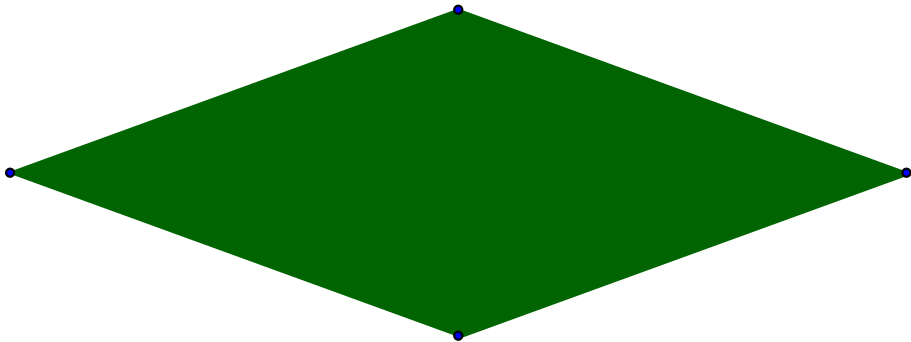
Des quadrilatères



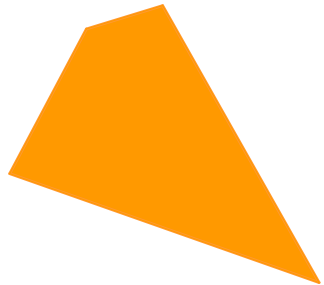
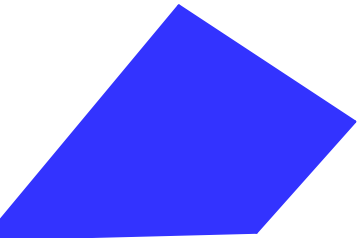
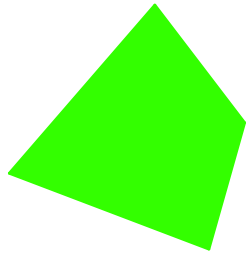
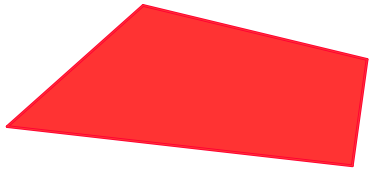




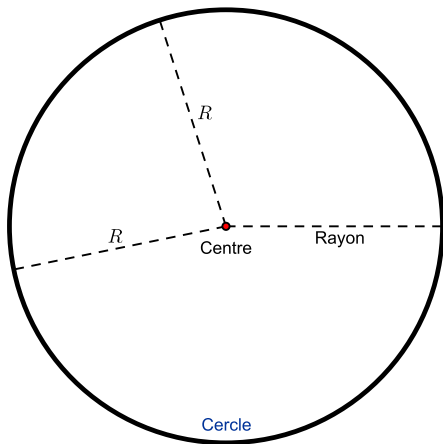
Parallélogramme



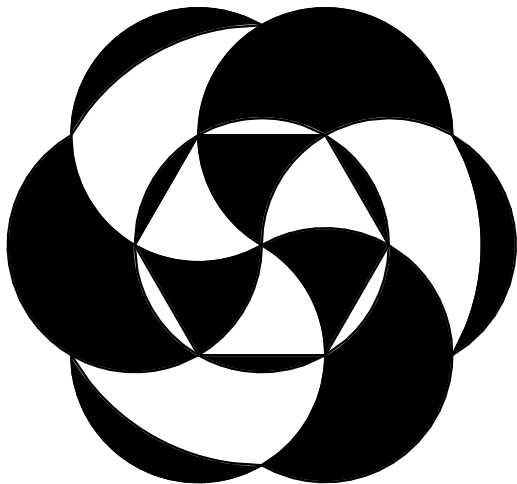
Losange



Le cercle



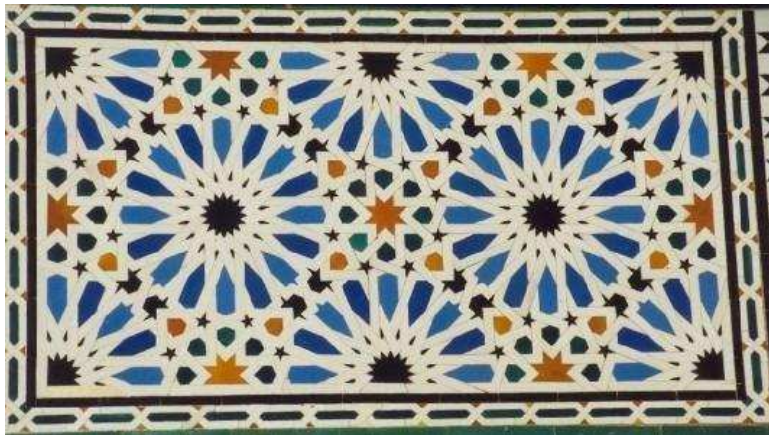
Le cercle Γ de centre O et de rayon $R \geq 0$ est l'ensemble des points dont la distance à O vaut R !



On peut fabriquer de belles figures avec des arcs de cercle !

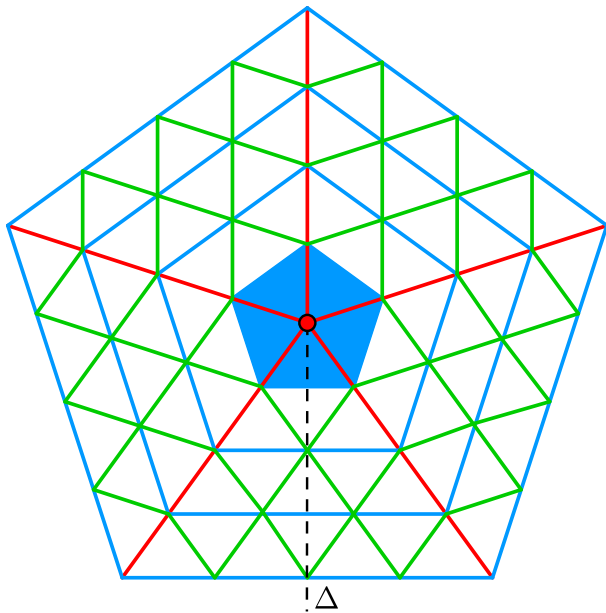
4. Pavages

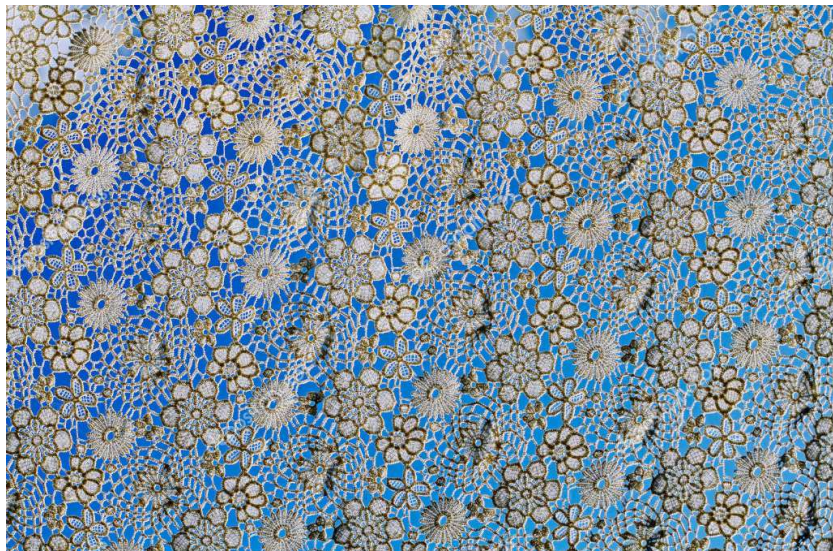
Quelques exemples

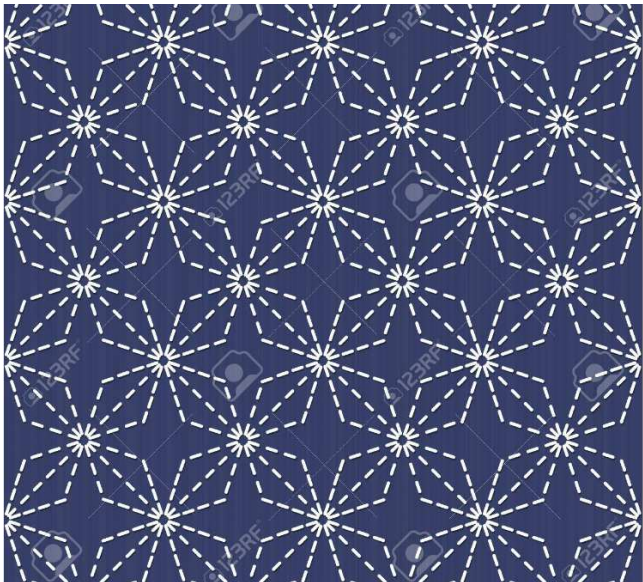




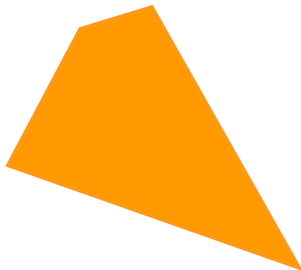
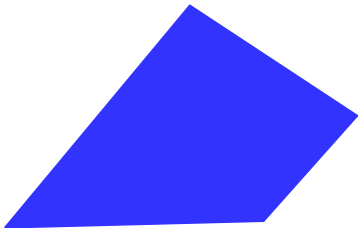
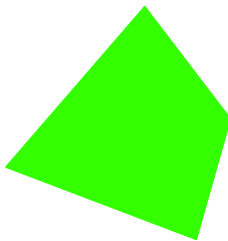
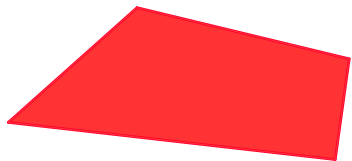


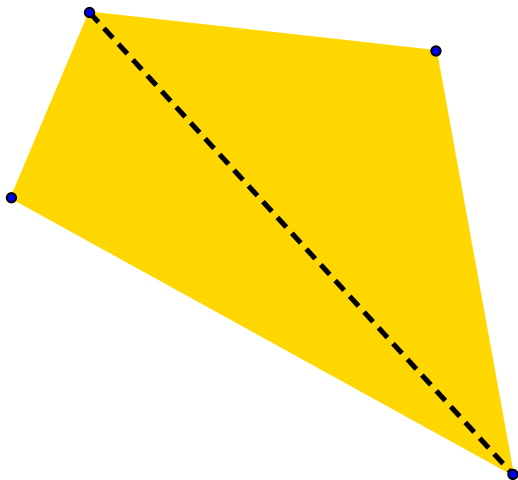




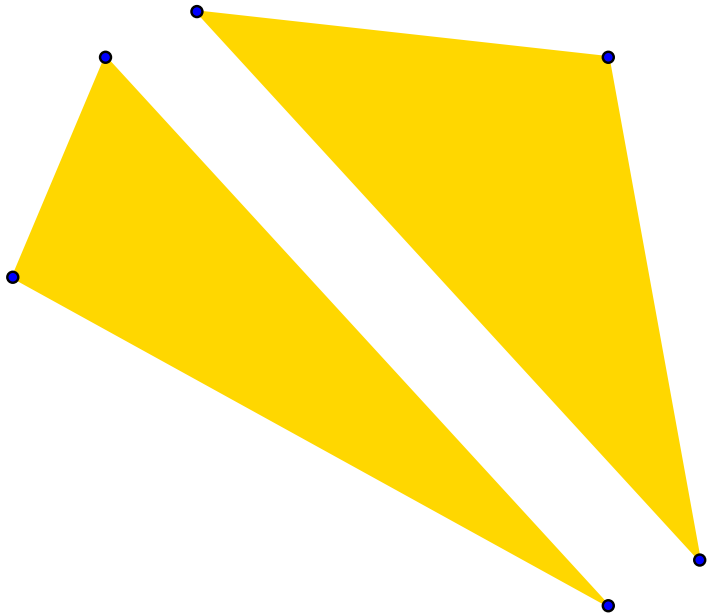


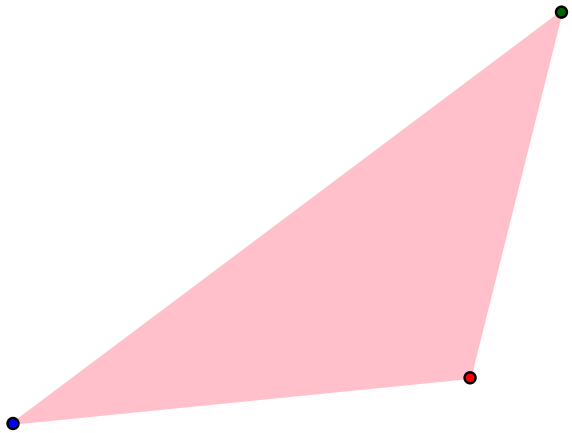
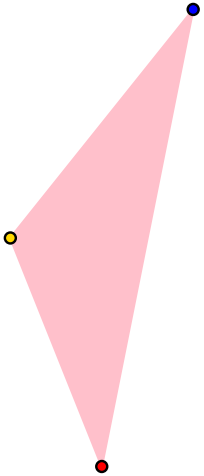
Découpage et recollement

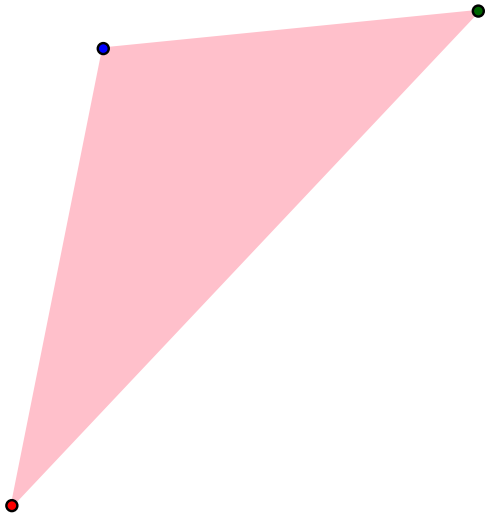
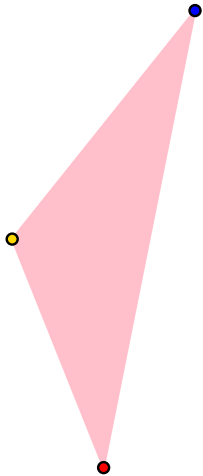


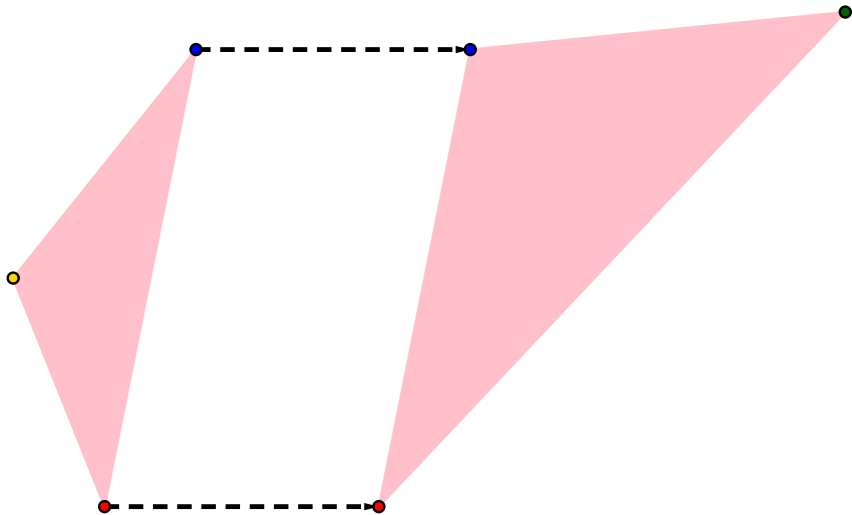


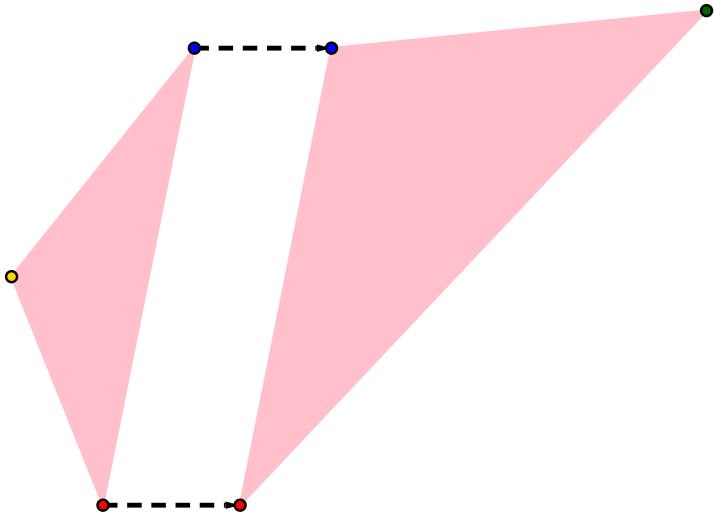
Quadrilatère quelconque et une diagonale

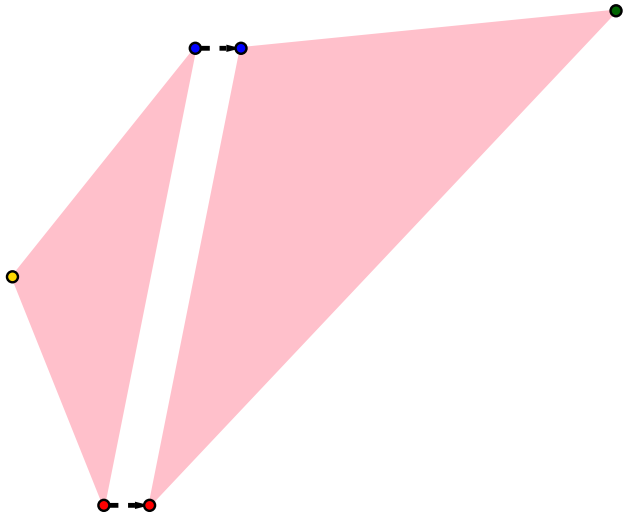


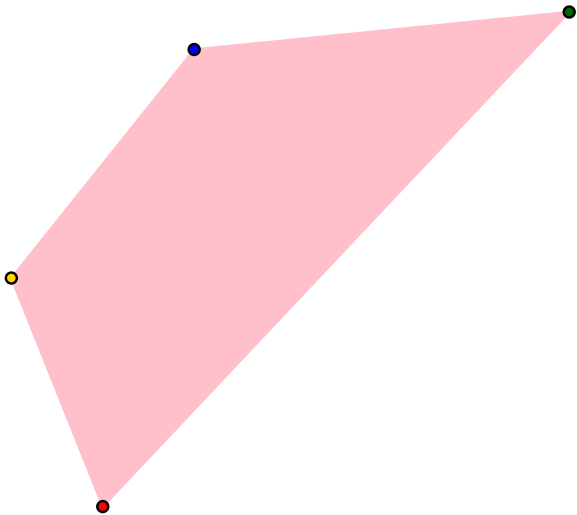


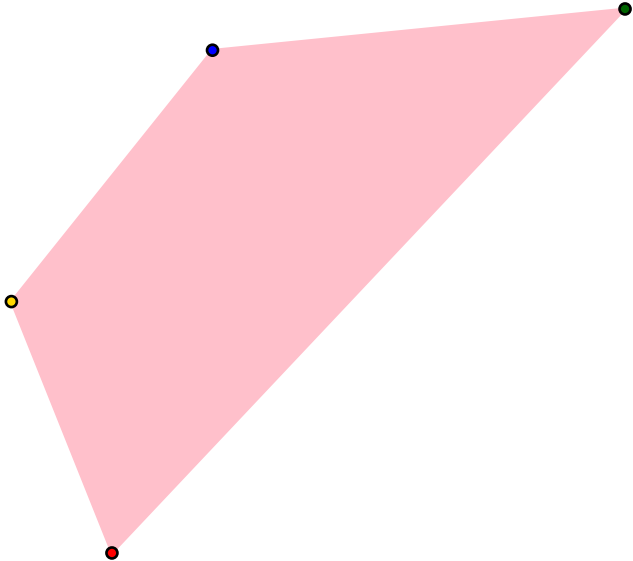


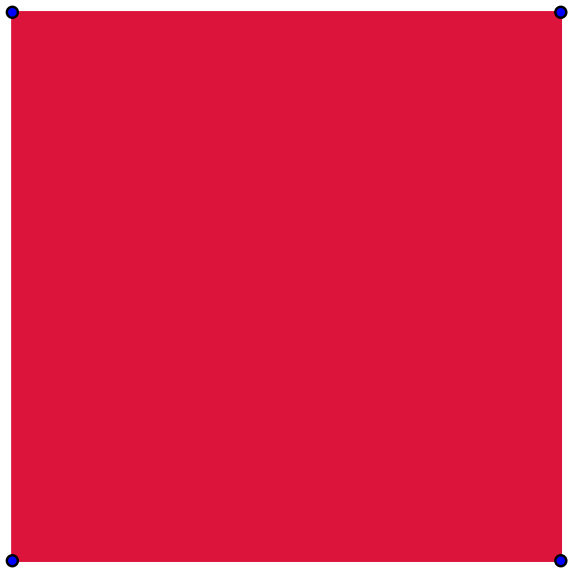


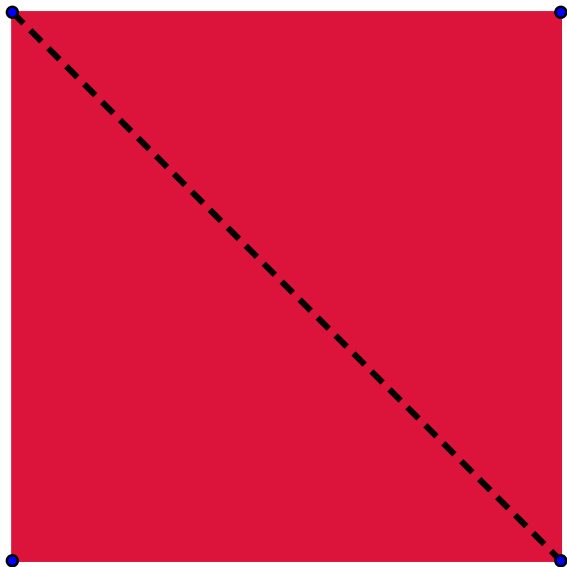




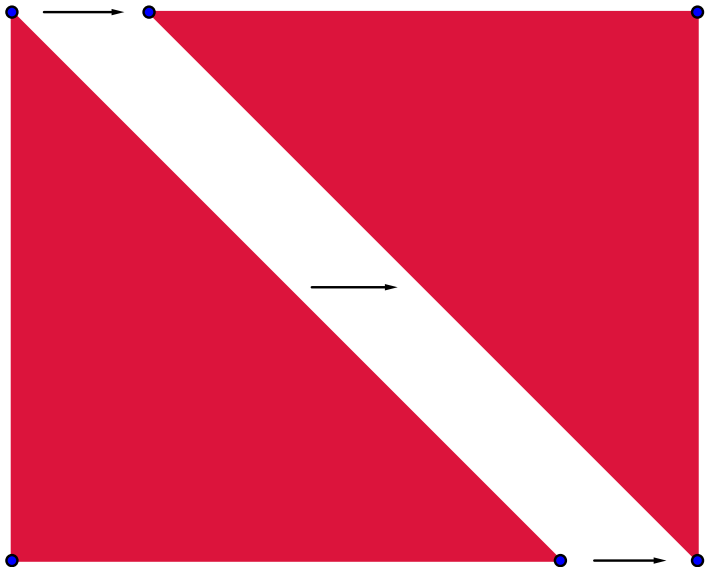


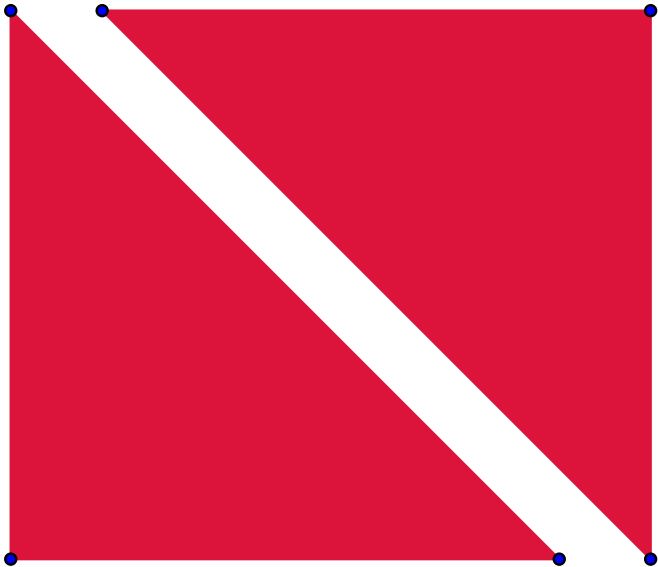


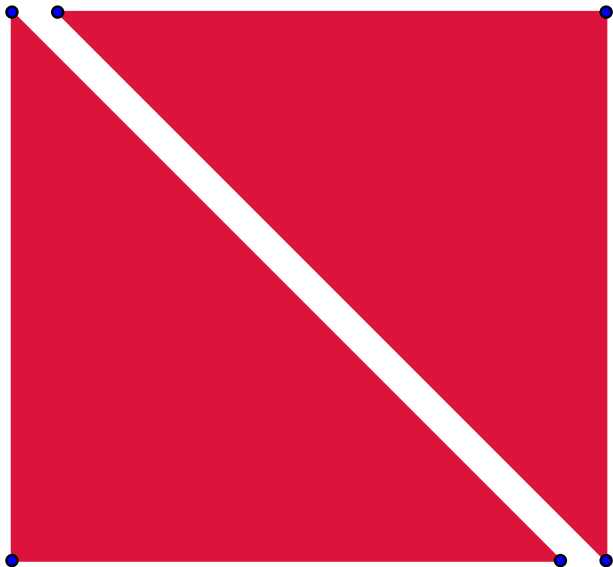




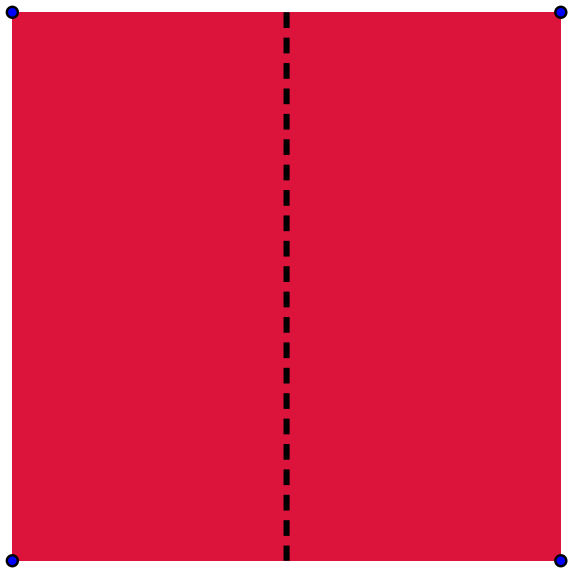




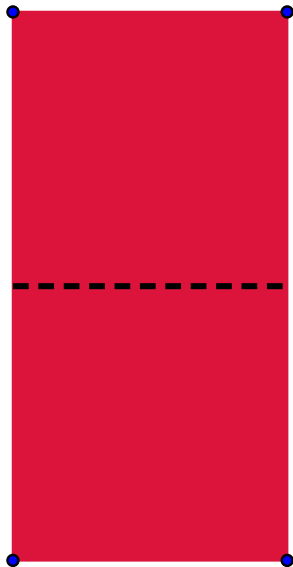
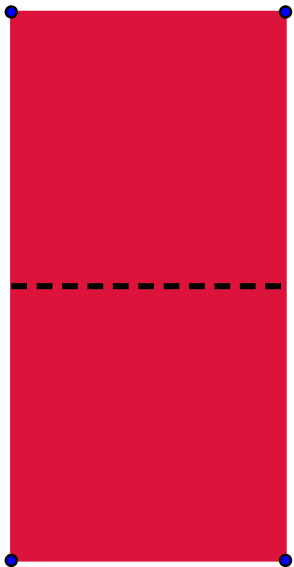


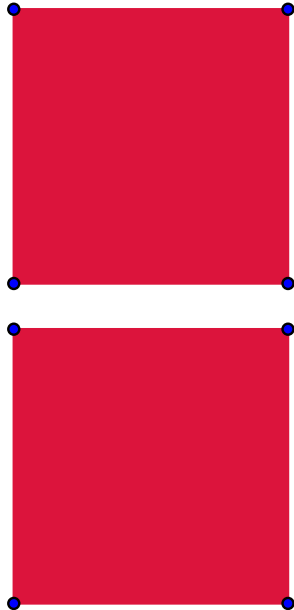
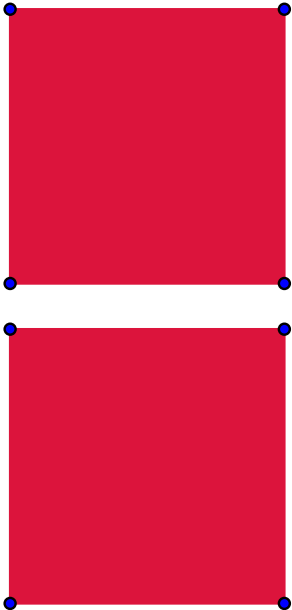


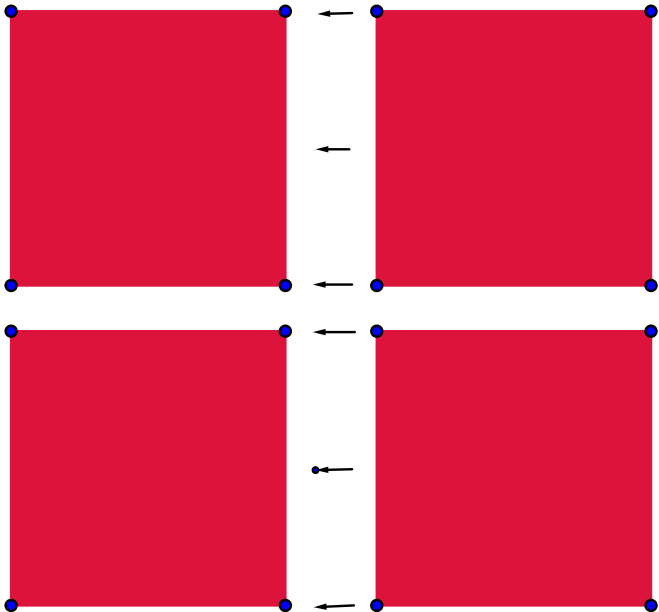


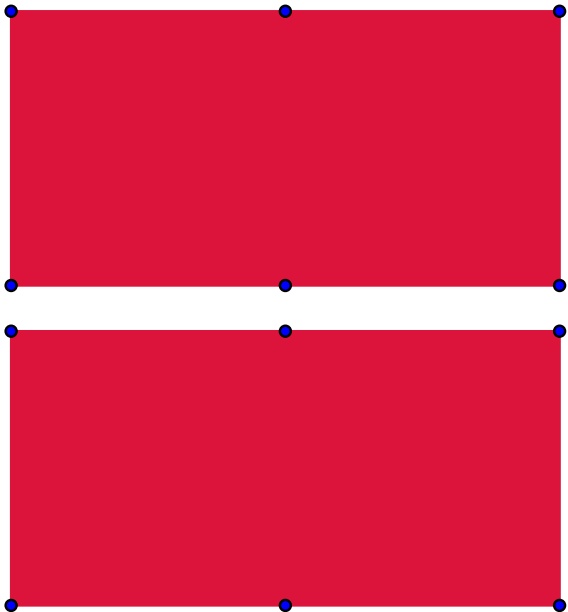


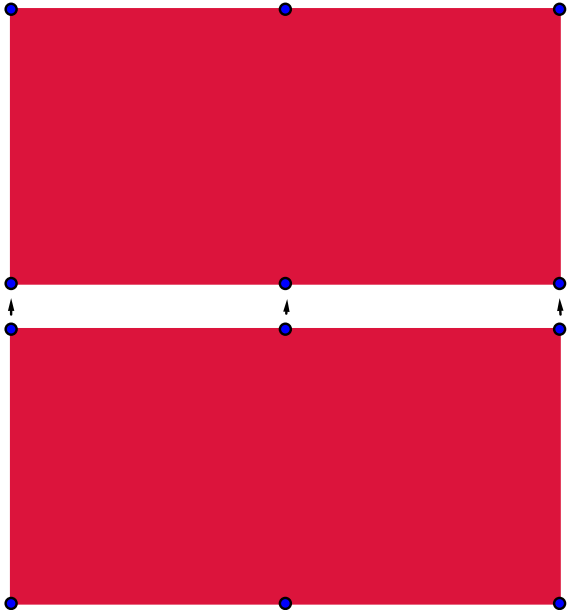






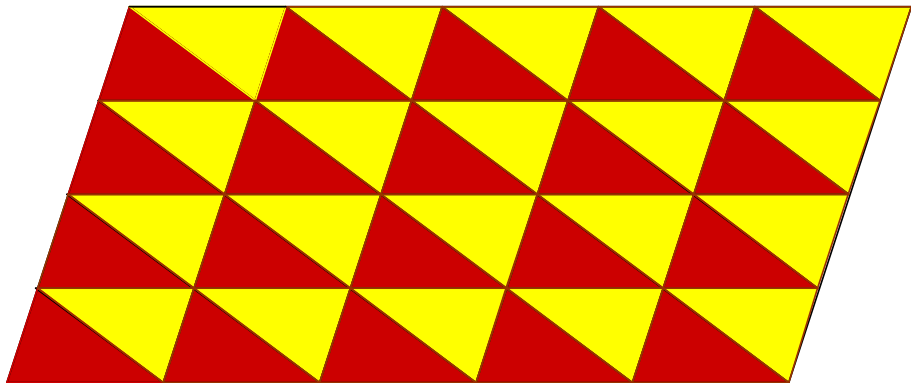




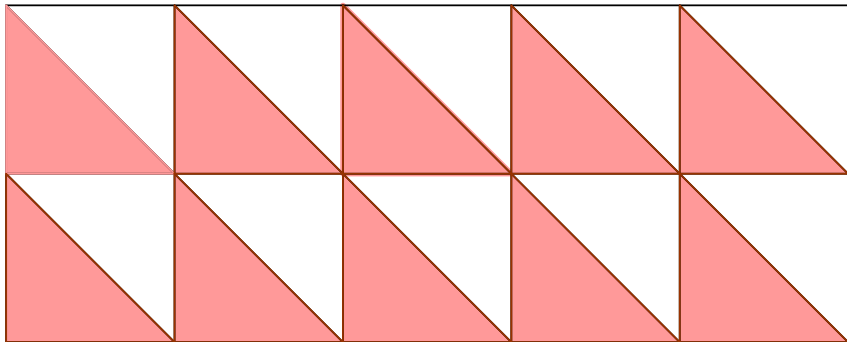




Triangle isocèle



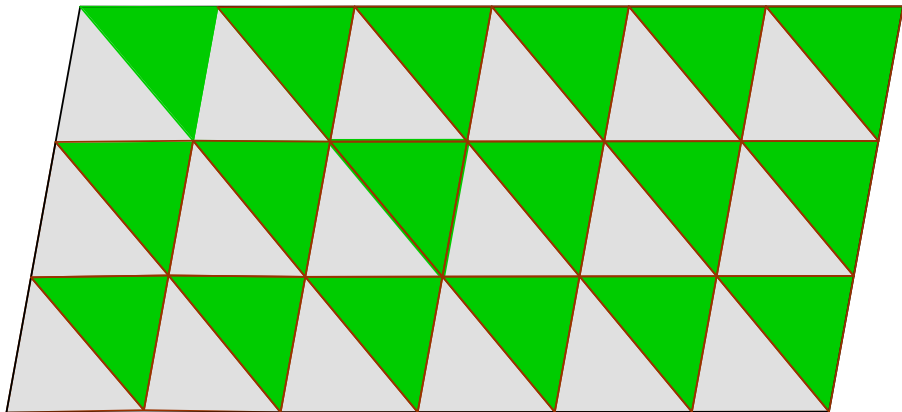
Triangle isocèle rectangle



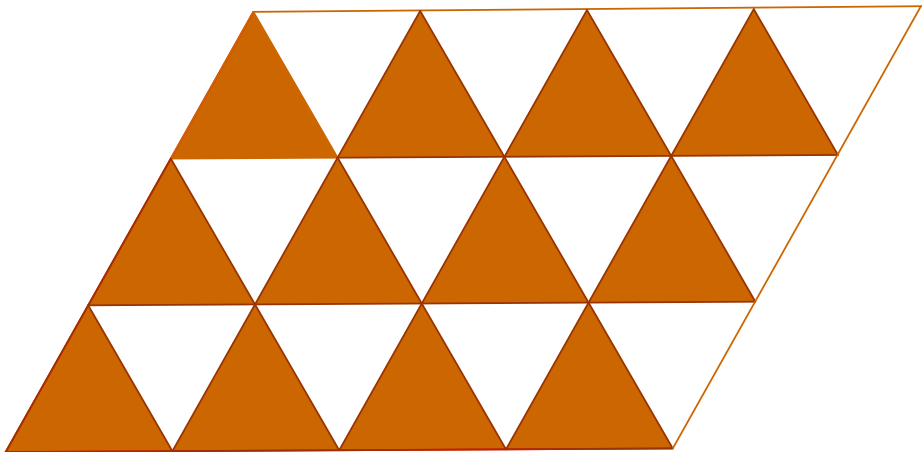
Triangle rectangle

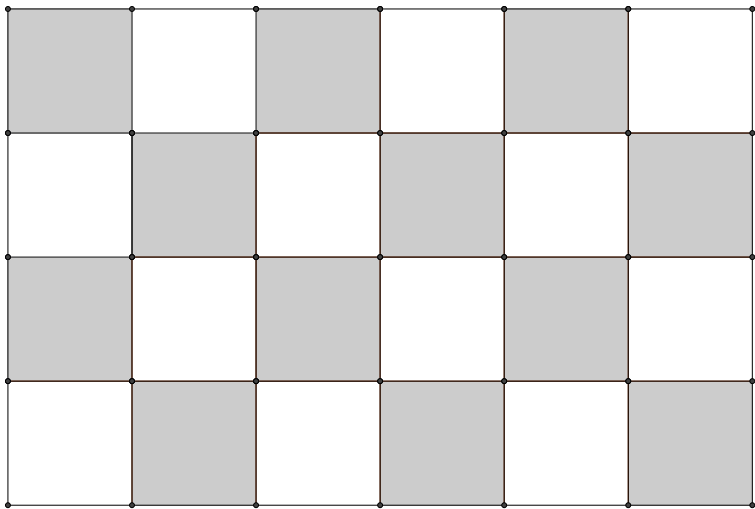


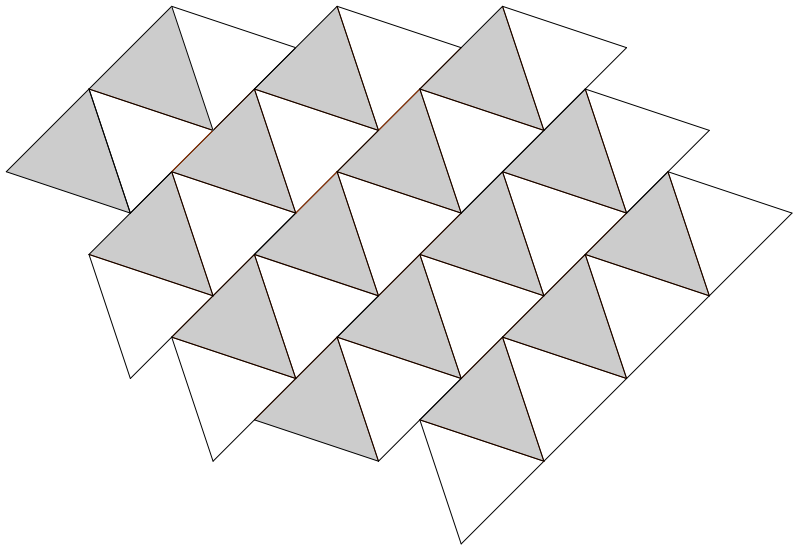
Triangle quelconque



Triangle équilatéral







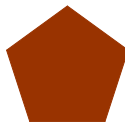
Quels sont les polygones réguliers qui pavent de façon périodique le plan euclidien \mathbb{E} ?



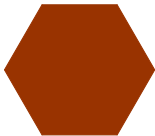
Triangle équilatéral



Carré



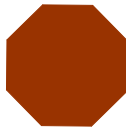
Pentagone



Hexagone



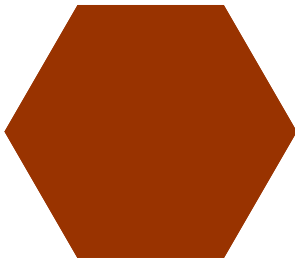
Heptagone



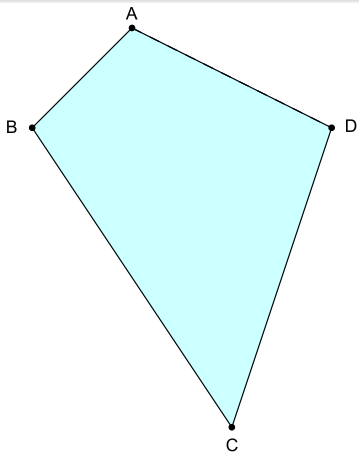
Octogone

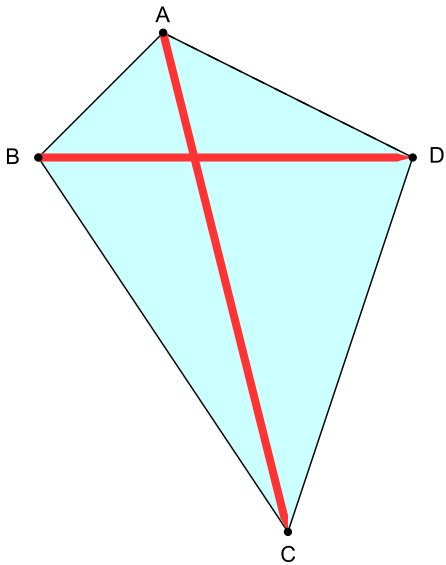
Polygones réguliers

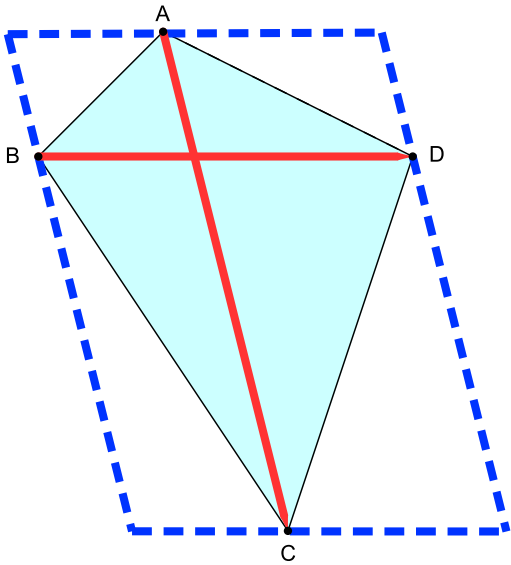
Polygones réguliers paveurs

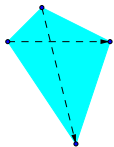


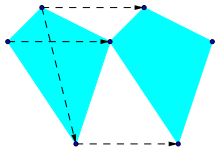
Carrelage par quadrilatères

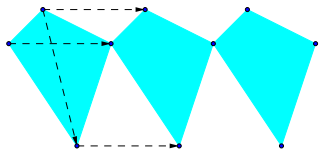


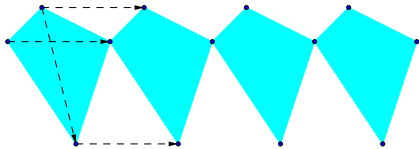


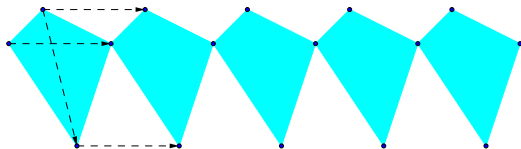


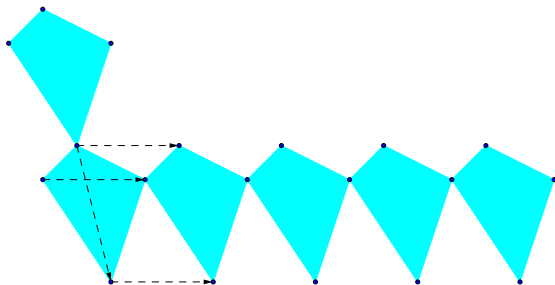


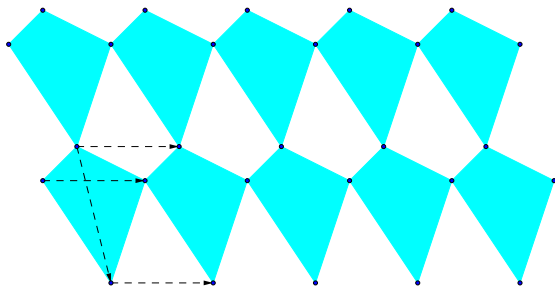


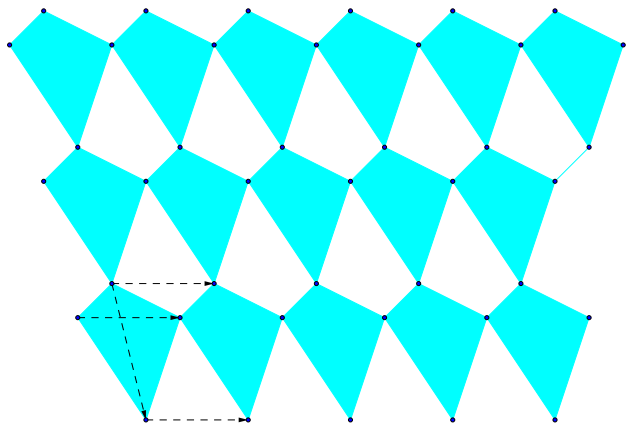


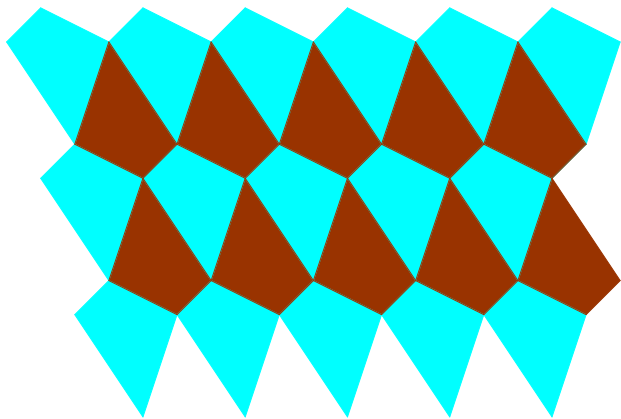


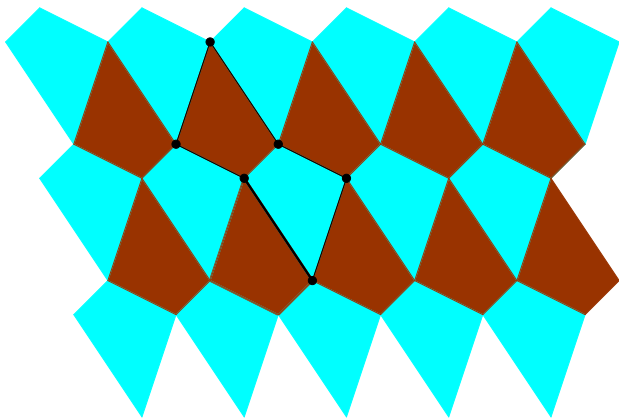




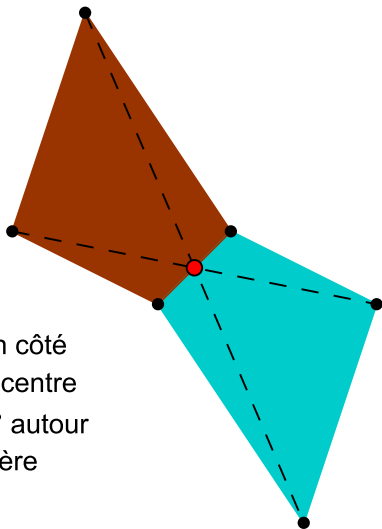


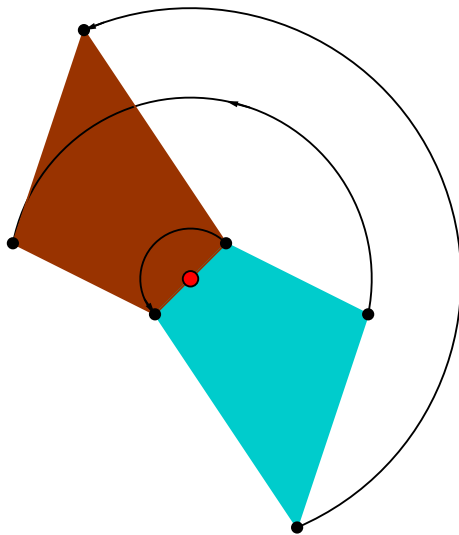






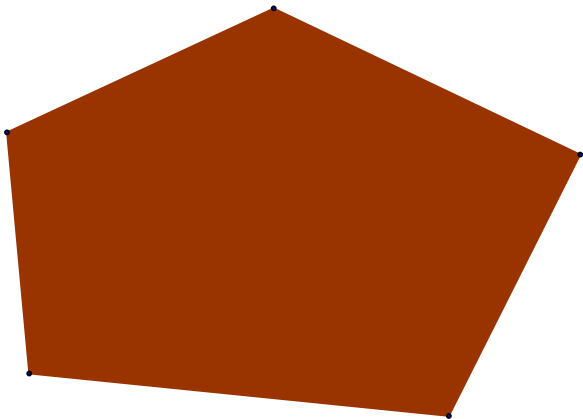
Deux quadrilatères collés sur un côté donnent un hexagone ayant un centre de symétrie. La rotation de 180° autour de ce centre amène le quadrilatère bleu sur le marron.

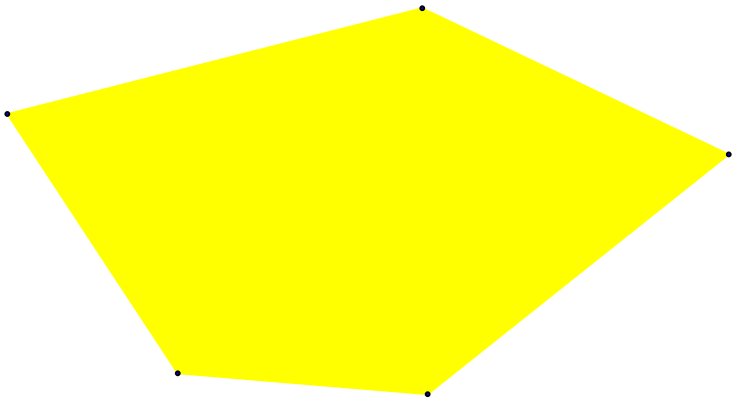


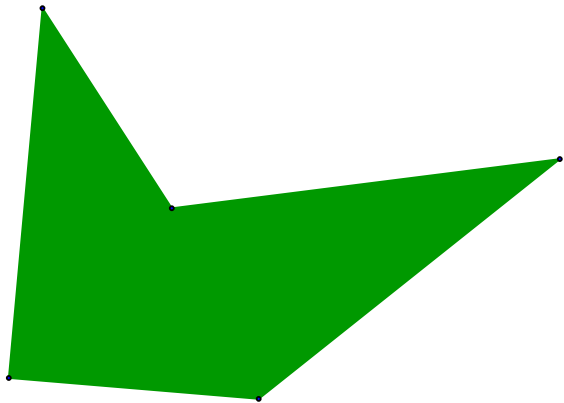


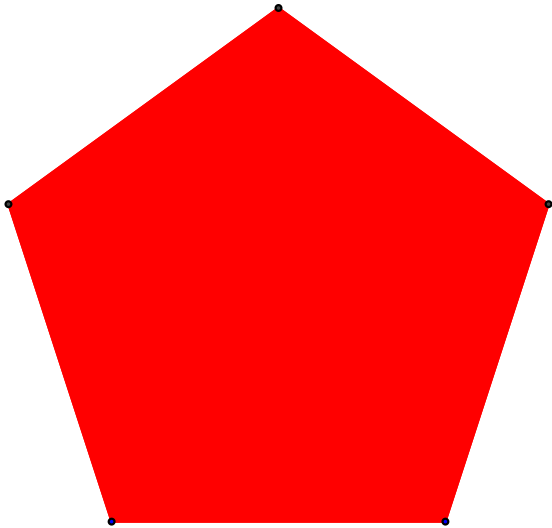
Hexagone avec un centre de symétrie

Des pentagones



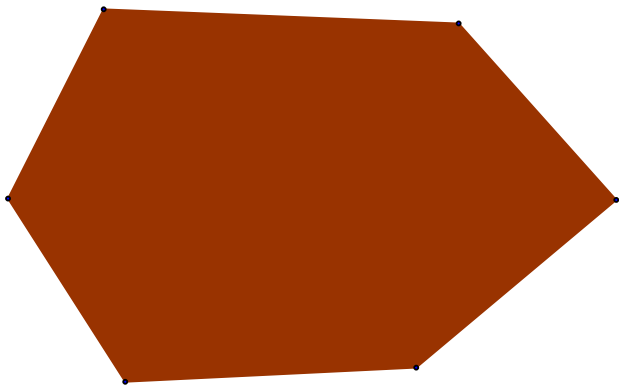


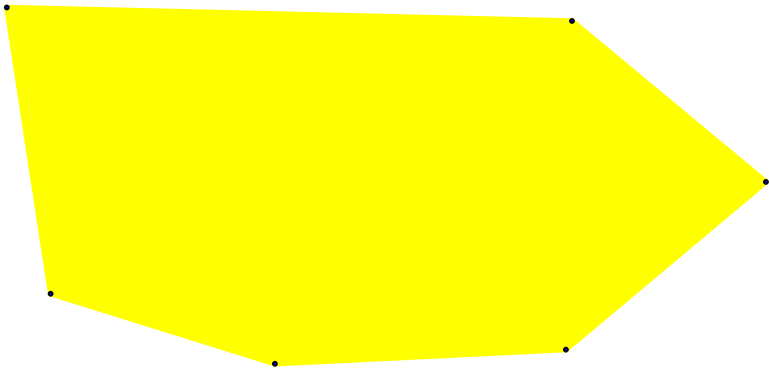


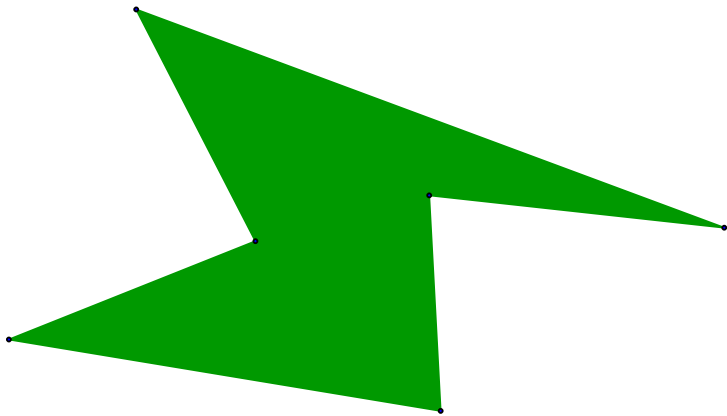


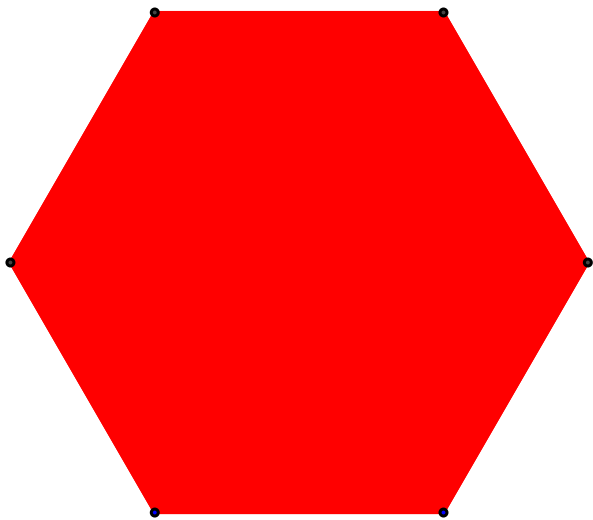
Pentagone régulier

Des hexagones









Hexagone régulier

