Referee's Report on the paper

The K-group of Substitutional Systems

by **Aziz El Kacimi and Rajagopalan Parthasarathy**

The paper is very interesting in my opinion because it gives us the tool how to compute (ordered) K-groups associated to aperiodic Cantor dynamical systems. The special case of Cantor minimal systems have been studied earlier with this problem in mind, and have been given a rather satisfactory treatment, but until the appearance of this paper we have not had a systematic way of attacking the much more general case of aperiodic Cantor systems —for example, aperiodic substitutional systems that are not primitive. The basic tool is as before ordered Bratteli diagram, but not necessarily simply ordered diagrams —the latter occurs naturally in the Cantor minimal case. In the basic Definition 1.10 of the paper it is condition (iii) that is crucial, as rightly stressed by the authors. The dynamical systems that are relevant for this condition to hold must have a family of (compatible) nested Kakutani-Rokhlin(K-R) partitions where the heights of the towers at level n (or equivalenly, the number of paths from the top vertex of the associated Bratteli diagram to any vertex at level n) go to infinity as n increases. In private communication with the authors they gave an argument for this, and I recommend that they include that argument in their paper. In fact, combining this with Proposition 3 and Corollary 4 of the paper "The Rokhlin lemma for homeomorphisms of a Cantor set" by Bezuglyi, Dooly and Medynets (*Proc. Amer. Math. Soc.* 133 (2005), 2957–2964), it follows that the Cantor systems that come under the purview of study in this paper are exactly the family of aperiodic Cantor systems. This I think should be pointed out explicitly, even in the introduction to the paper. The authors show how one can compute the K-groups for these systems using their modified Bratteli diagram model, and they specialize to substitution systems to demonstrate their method.

The paper is very well written and contains original new results, and I highly recommend that it be published in your journal. (Please find a short list below with a few comments, and pointing out misprints, etc.)

Comments:

- page 2, line 14-: "$i < j$" instead of "$1 < j$".

- page 4, section 1.8 and 19: give an argument for what you claim, namely that if the heights of the K-R towers go to infinity, then the interval segments behave as you claim. Also, refer to the result in the paper by Bezyglui, Dooly and Medynets, mentioned in the referee report.

- page 6, line 1- and page 7, line 17 (twice) and page 12, lines 8-, 16- (twice): "letter" instead of "alphabet".

- page 9, line 22: "Even though" instead of "Eventhough".

- page 11, line 5: "...defined as in (1.10) and (1.11)." instead of "...defined as in (1.10)."

- page 11, line 11-: "Herman" instead of "Hermann".