

5. Formules de développements limités

Développements limités usuels (au voisinage de 0)

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + o(x^n) = \sum_{k=0}^n \frac{x^k}{k!} + o(x^n)$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \cdot \frac{x^{2n}}{(2n)!} + o(x^{2n+1}) = \sum_{k=0}^n (-1)^k \frac{x^{2k}}{(2k)!} + o(x^{2n+1})$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \cdot \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) = \sum_{k=0}^n (-1)^k \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2})$$

$$\tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5 + \frac{17}{315}x^7 + o(x^8)$$

$$\operatorname{ch} x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + o(x^{2n+1}) = \sum_{k=0}^n \frac{x^{2k}}{(2k)!} + o(x^{2n+1})$$

$$\operatorname{sh} x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + o(x^{2n+2}) = \sum_{k=0}^n \frac{x^{2k+1}}{(2k+1)!} + o(x^{2n+2})$$

$$\operatorname{th} x = x - \frac{x^3}{3} + \frac{2}{15}x^5 - \frac{17}{315}x^7 + o(x^8)$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \cdot \frac{x^n}{n} + o(x^n) = \sum_{k=1}^n (-1)^{k+1} \frac{x^k}{k} + o(x^n)$$

$$\begin{aligned} (1+x)^\alpha &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \cdots + \frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n + o(x^n) \\ &= \sum_{k=0}^n \binom{\alpha}{k} x^k + o(x^n) \end{aligned}$$

$$\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-1)^n x^n + o(x^n) = \sum_{k=0}^n (-1)^k x^k + o(x^n)$$

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + o(x^n) = \sum_{k=0}^n x^k + o(x^n)$$

$$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{1}{8}x^2 - \cdots + (-1)^{n-1} \cdot \frac{1 \cdot 1 \cdot 3 \cdot 5 \cdots (2n-3)}{2^n n!} x^n + o(x^n)$$

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{x}{2} + \frac{3}{8}x^2 - \cdots + (-1)^n \cdot \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2^n n!} x^n + o(x^n)$$

$$\arccos x = \frac{\pi}{2} - x - \frac{1}{2} \frac{x^3}{3} - \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} - \cdots - \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$$

$$\arcsin x = x + \frac{1}{2} \frac{x^3}{3} + \frac{1 \cdot 3}{2 \cdot 4} \frac{x^5}{5} + \cdots + \frac{1 \cdot 3 \cdot 5 \cdots (2n-1)}{2 \cdot 4 \cdot 6 \cdots (2n)} \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$$

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \cdot \frac{x^{2n+1}}{2n+1} + o(x^{2n+2})$$