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- Intersection cohomology of $\mathbb{S}^1$-actions.
- The Euler class for flows of isometries.

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- Euler y un balón de fútbol.
- A six dimensional compact symplectic solvmanifold without Kähler structure.
- Cosymplectic reduction for singular momentum maps.
- Fuzzy Filters.
- Una nota sobre convergencia en espacios topológicos fuzzy.

Addendum to "Refinement invariance of intersection (co)homologies"

In a previous work we proved the refinement invariance of several intersection (co)homologies existing in the literature. Specifically, we worked with a refinement $f\colon (X, \mathcal{S}) \to (X, \mathcal{T})$ between two CS-sets where the strata of $\mathcal{S}$ were embedded in the strata of $\mathcal{T}$. However, in this paper, we establish that this embedding condition is not a requirement for the refinement invariance property.

Homotopy truncations of homotopically stratified spaces

Intersection homology of Goresky and MacPherson can be defined from the Deligne sheaf, obtained from truncations of complexes of sheaves. As intersection homology is not the homology of a particular space, the search for a family of spaces whose homologies have properties analogous to intersection homology has developed. For some stratified spaces, M. Banagl has introduced such a family by using a topological truncation: the original link is replaced by a truncation of its homological Moore resolution. In this work, we study the dual approach in the Bernstein-Hilton sense : we consider the stratified space obtained by replacing the original link by a Postnikov approximation. The main result is that our construction restores the space constructed by Gajer to establish an intersection Dold-Thom theorem. We are conducting this study within the general framework of Quinn's homotopically stratified spaces.

Perverse homotopy groups

As Goresky and MacPherson intersection homology is not the homology of a space, there is no preferred candidate for intersection homotopy groups. Here, they are defined as the homotopy groups of a simplicial set which P. Gajer associates to a couple $(X, \overline{p})$ of a filtered space and a perversity. We first establish some basic properties for the intersection fundamental groups, as a Van Kampen theorem.

For general intersection homotopy groups on Siebenmann CS sets, we prove a Hurewicz theorem between them and the Goresky and MacPherson intersection homology. If the CS set and its intrinsic stratification have the same regular part, we establish the topological invariance of the $\overline{p}$-intersection homotopy groups. Several examples justify the hypotheses made in the statements. Finally, intersection homotopy groups also coincide with the homotopy groups of the topological space itself, for the top perversity on a connected, normal Thom-Mather space.

A reasonable notion of dimension for singular intersection homology

M. Goresky and R. MacPherson intersection homology is also defined from the singular chain complex of a filtered space by H. King, with a key formula to make selections among singular simplexes. This formula needs a notion of dimension for subspaces S of a Euclidean simplex, which is usually taken as the smallest dimension of the skeletta containing S. Later, P. Gajer employed another dimension based on the dimension of polyhedra containing S. This last one allows traces of pullbacks of singular strata in the interior of the domain of a singular simplex.
In this work, we prove that the two corresponding intersection homologies are isomorphic for Siebenmann's CS sets. In terms of King's paper, this means that polyhedral dimension is a "reasonable" dimension. The proof uses a Mayer-Vietoris argument which needs an adaptated subdivision. With the polyhedral dimension, that is a subtle issue. General position arguments are not sufficient and we introduce strong general position. With it, a stability is added to the generic character and we can do an inductive cutting of each singular simplex. This decomposition is realized with pseudo-barycentric subdivisions where the new vertices are not barycentres but close points of them.

Simplicial intersection homology revisited

Intersection homology is defined for simplicial, singular and PL chains and it is well known that the three versions are isomorphic for a full filtered simplicial complex. In the literature, the isomorphism, between the singular and the simplicial situations of intersection homology, uses the PL case as an intermediate.
Here we show directly that the canonical map between the simplicial and the singular intersection chains complexes is a quasi-isomorphism. This is similar to the classical proof for simplicial complexes, with an argument based on the concept of residual complex and not on skeletons.
This parallel between simplicial and singular approaches is also extended to the intersection blown-up cohomology that we introduced in a previous work. In the case of an orientable pseudomanifold, this cohomology owns a Poincaré isomorphism with the intersection homology, for any coefficient ring, thanks to a cap product with a fundamental class. So, the blown-up intersection cohomology of a pseudomanifold can be computed from a triangulation.
Finally, we introduce a blown-up intersection cohomology for PL spaces and prove that it is isomorphic to the singular one.

Refinement invariance of intersection (co)homologies

We study the refinement invariance of several intersection (co)homologies existing in the literature. These (co)homologies have been introduced in order to establish the Poincaré Duality in various contexts. We found the classical topological invariance of the intersection homology and also various refinement invariance results already proved.

Intersection homology with coefficients in a field restores Poincaré duality for some spaces with singularities, as pseudomanifolds. But, with coefficients in a ring, the behaviours of manifolds and pseudomanifolds are different. This work is an overview, with proofs and explicit examples, of various possible situations with their properties.
We first set up a duality, defined from a cap product, between two intersection cohomologies: the first one arises from a linear dual and the second one from a simplicial blow up. Moreover, from this property, Poincaré duality in intersection homology looks like the Poincaré-Lefschetz duality of a manifold with boundary. Besides that, an investigation of the coincidence of the two previous cohomologies reveals that the only obstruction to the existence of a Poincaré duality is the homology of a well defined complex. This recovers the case of the peripheral sheaf introduced by Goresky and Siegel for compact PL-pseudomanifolds. We also list a series of explicit computations of peripheral intersection cohomology. In particular, we observe that Poincaré duality can exist in the presence of torsion in the "critical degree" of the intersection homology of the links of a pseudomanifold.

Poincaré duality, cap products and Borel-Moore Intersection Homology

Using a cap product, we construct an explicit Poincaré isomorphism between the blown-up intersection cohomology and the Borel-Moore intersection homology, for any commutative ring of coefficients and second-countable, oriented pseudomanifold.

Blown-up intersection cochains and Deligne's sheaves

In a series of papers the authors introduced the so-called blown-up intersection cochains. These cochains are suitable to study products and cohomology operations of intersection cohomology of stratified spaces. The aim of this paper is to prove that the sheaf versions of the functors of blown-up intersection cochains are realizations of Deligne's sheaves. This proves that Deligne's sheaves can be incarnated at the level of complexes of sheaves by soft sheaves of perverse differential graded algebras. We also study Poincaré and Verdier dualities of blown-up intersections sheaves with the use of Borel-Moore chains of intersection.

For a Riemannian foliation $\mathcal{F}$ on a compact manifold M , J. A. Álvarez López proved that the geometrical tautness of $\mathcal{F}$, that is, the existence of a Riemannian metric making all the leaves minimal submanifolds of M, can be characterized by the vanishing of a basic cohomology class $\boldsymbol{\kappa}_M \in H^1(M/\mathcal{F})$ (the Álvarez class). In this work we generalize this result to the case of a singular Riemannian foliation $\mathcal{K}$ on a compact manifold X. In the singular case, no bundle-like metric on X can make all the leaves of $\mathcal{K}$ minimal. In this work, we prove that the çlvarez classes of the strata can be glued in a unique global çlvarez class $\boldsymbol{\kappa}_X \in H^1(X/\mathcal{K})$. As a corollary, if X is simply connected, then the restriction of $\mathcal{K}$ to each stratum is geometrically taut, thus generalizing a celebrated result of E. Ghys for the regular case.

Lefschetz duality for intersection (co)homology

We prove the Lefschetz duality for intersection (co)homology in the framework of ∂-pesudomanifolds. We work with general perversities and without restriction on the coefficient ring.

Blown-up intersection cohomology

In previous works, we have introduced the blown-up intersection cohomology and used it to extend Sullivan's minimal models theory to the framework of pseudomanifolds, and to give a positive answer to a conjecture of M. Goresky and W. Pardon on Steenrod squares in intersection homology. In this paper, we establish the main properties of this cohomology. One of its major feature is the existence of cap and cup products for any filtered space and any commutative ring of coefficients, at the cochain level. Moreover, we show that each stratified map induces an homomorphism between the blown-up intersection cohomologies, compatible with the cup and cap products. We prove also its topological invariance in the case of a pseudomanifold with no codimension one strata. Finally, we compare it with the intersection cohomology studied by G. Friedman and J.E. McClure. A great part of our results involves general perversities, defined independently on each stratum, and a tame intersection homology adapted to large perversities.

Singular factorization of a cap-product

In the case of a compact orientable pseudomanifold, a well-known theorem of M. Goresky and R. MacPherson says that the cap-product with a fundamental class factorizes through the intersection homology groups. In this work, we show that this classical cap-product is compatible with a cap-product in intersection (co)-homology, that we have previously introduced. As a corollary, for any commutative ring of coefficients, the existence of a classical Poincar? duality isomorphism is equivalent to the existence of an isomorphism between the intersection homology groups corresponding to the zero and the top perversities. Our results answer a question asked by G. Friedman.

Poincaré? duality with cap products in intersection homology

For having a Poincaré? duality via a cap product between the intersection homology of a paracompact oriented pseudomanifold and the cohomology given by the dual complex, G. Friedman and J. E. McClure need a coefficient field or an additional hypothesis on the torsion. In this work, by using the classical geometric process of blowing-up, adapted to a simplicial setting, we build a cochain complex which gives a Poincaré? duality via a cap product with intersection homology, for any commutative ring of coefficients. We prove also the topological invariance of the blown-up intersection cohomology with compact supports in the case of a paracompact pseudomanifold with no codimension one strata. This work is written with general perversities, defined on each stratum and not only in function of the codimension of strata. It contains also a tame intersection homology, suitable for large perversities.

Topological invariance of the intersection homology of a pseudomanifold without codimension one strata, proved by M. Goresky and R. MacPherson, is one of the main properties of this homology. This property is true for strata codimension depending perversities with some growth conditions verifying $\overline{p}(1) = \overline{p}(2) = 0$. H. King reproves this invariance by associating an intrinsic pseudomanifold X^+ to any pseudomanifold X. His proof consists of an isomorphism between the associated intersection homologies $H_*^{\overline{p}}(X) \cong H_*^{\overline{p}}\left(X^+\right)$, for any perversity $\overline{p}$ with the same growth conditions verifying $\overline{p}(1) \geqslant 0$.
In this work, we prove a certain topological invariance within the framework of strata depending perversities, $\overline{p}$, which corresponds to the classical topological invariance if $\overline{p}$ is a GM-perversity. We also extend it to the tame intersection homology, a variation of the intersection homology, particularly suited for "large" perversities, if there is no singular strata on X becoming regular in X^+. In particular, under the above conditions, the intersection homology and the tame intersection are invariant under a refinement of the stratification.

Steenrod squares on Intersection cohomology and a conjecture of M. Goresky and W. Pardon

We prove a conjecture raised by M. Goresky and W. Pardon, concerning the range of validity of the perverse degree of Steenrod squares in intersection cohomology. This answer turns out of importance for the definition of characteristic classes in the framework of intersection cohomology. For this purpose, we present a construction of cup_i-products on the cochain complex, built on the blow-up of some singular simplices and introduced in a previous work. We extend to this setting the classical properties of the associated Steenrod squares, including Adem and Cartan relations, for any generalized perversities. In the case of a pseudomanifold, we prove that our definition coincides with M. Goresky's definition. Several examples of concrete computation of perverse Steenrod squares are given, including the case of isolated singularities and, more especially, we describe the Steenrod squares on the Thom space of a vector bundle, in function of the Steenrod squares of the basis and the Stiefel-Whitney classes. We detail also an example of a non trivial square, $\mathrm{Sq}^2 \colon H^{\star}_{\overline{p}} \to H^{\star}_{\overline{p}+2}$, whose information is lost if we consider it as values in $H^{2}_{\overline{p}}$, showing the interest of the Goresky and Pardon's conjecture.

Let X be a pseudomanifold. In this text, we use a simplicial blow-up to define a cochain complex whose cohomology with coefficients in a field, is isomorphic to the intersection cohomology of X, introduced by M. Goresky and R. MacPherson. We do it simplicially in the setting of a filtered version of face sets, also called simplicial sets without degeneracies, in the sense of C.P. Rourke and B.J. Sanderson. We define perverse local systems over filtered face sets and intersection cohomology with coefficients in a perverse local system. In particular, as announced above when X is a pseudomanifold, we get a perverse local system of cochains quasi-isomorphic to the intersection cochains of Goresky and MacPherson, over a field. We show also that these two complexes of cochains are quasi-isomorphic to a filtered version of Sullivan's differential forms over the field $\mathbb{Q}$. In a second step, we use these forms to extend Sullivan's presentation of rational homotopy type to intersection cohomology. For that, we construct a functor from the category of filtered face sets to a category of perverse commutative differential graded $\mathbb{Q}$-algebras (cdga's) due to Hovey. We establish also the existence and unicity of a positively graded, minimal model of some perverse cdga's, including the perverse forms over a filtered face set and their intersection cohomology. Finally, we prove the topological invariance of the minimal model of a PL-pseudomanifold whose regular part is connected, and this theory creates new topological invariants. This point of view brings a definition of formality in the intersection setting and examples are given. In particular, we show that any nodal hypersurface in $\mathbb{CP}(4)$, is intersection-formal.

For a stratified pseudomanifold X, we have the de Rham Theorem $IH^{\star}_{\overline{p}}(X) = IH^{\overline{t}-\overline{p}}_{\star}(X)$, for a perversity $\overline{p}$ verifying $\overline{0} \leqslant \overline{p} \leqslant \overline{t}$, where $\overline{t}$ denotes the top perversity. We extend this result to any perversity $\overline{p}$. In the direction cohomology $\longmapsto$ homology, we obtain the isomorphism $IH^{\star}_{\overline{p}}(X) = IH^{\overline{t}-\overline{p}}_{\star}\left(X, X_{\overline{p}}\right)$, where $X_{\overline{p}} = \bigcup_{\substack{S \preceq S_1 \\ \overline{p}(S_1)<0}} S = \bigcup_{\overline{p}(S)<0} \overline{S}$. In the direction homology $\longmapsto$ cohomology, we obtain the isomorphism $IH^{\overline{p}}_{\star}(X) = IH^{\star}_{\max(\overline{0},\overline{t}-\overline{p})}(X)$. In our paper stratified pseudomanifolds with one-codimensional strata are allowed.

Cohomologie d'intersection mod?r?e. Un Th?or?me de deRham.

With a singular space K there is associated a differential graded module of polynomial differential forms $IT_{\overline{p}}^{\star,\star}(K)$ together with a filtration $IT_{\overline{p}}^{\star,q}(K) \subset IT_{\overline{p}}^{\star,q+1}(K)$ in each degree $\star$. $IT_{\overline{p}}^{\star,q}(K)$ is a graded module over the subring of the rationals $\mathbb{Q}_q = \mathbb{Z}[\frac{1}{2}, \frac{1}{3}, \ldots, \frac{1}{q}]$. These modules are defined for any stratified pseudomanifold K and for any perversity $\overline{p}$. It is proved that the cohomology of such a differential module $IT_{\overline{p}}^{\star,q}(K)$ is isomorphic to the intersection cohomology $IH_{\overline{p}}^{\star}(K;\mathbb{Q}_q)$. The construction of $IT_{\overline{p}}^{\star,\star}(K)$ is based on the deRham complex of Cenkl and Porter when applied to a desingularization of K.

Homological properties of stratified spaces.

We establish a de Rham Theorem $IH^{\star}_{\overline{p}}(X) = IH^{\overline{t}-\overline{p}}_{\star}(X)$, for a general perversity $\overline{p}$ verifying $\overline{0} \leqslant \overline{p} \leqslant \overline{t}$, where $\overline{t}$ denotes the top perversity. We work in the framework of unfoldable pseudomanifolds.

Vari?t?s homologiques et homologie d'intersection.

In this Note, we prove that each stratified pseudomanifold A satisfying:

(a) the intersection homology groups $IH_{\star}^{\overline{p}}(A)$ are isomorphic for each loose perversity $\overline{p}$;

(b) each stratum S possess a tubular neighborhood U_S, whose homological monodromy is trivial, is a homology manifold. This generalizes a result of King, where the triviality of the U_S themselves was required.

$\mathcal{L}^2$-cohomologie des espaces stratifi?s.

The first results relating intersection homology with $\mathcal{L}^2$-cohomology were found by Cheeger, Goresky and MacPherson. The first spaces considered were the compact stratified pseudomanifolds with isolated singularities. Later, Nagase extended this result to any compact stratified space A possessing a Cheeger type riemannian metric μ. The proof of the isomorphism $H^{\star}_{(2)}(A - \Sigma; \mu) \cong IH^{\overline{p}}_{\star}(A)$ uses the axiomatic characterization of the intersection homology of the authors. In this work we show how to realize this isomorphism by the usual integration of differential forms on simplexes. The main tool used is the blow up of A into a smooth manifold, introduced in the previous work of the authors. We also prove that any stratified space possesses a Cheeger type riemannian metric.

Th?or?me de de Rham pour les vari?t?s stratifi?es.

We consider a stratified space endowed with a Thom-Mather system. We prove that, for any classical perversity, the usual integration establishes an isomorphism between intersection homology and deRham intersection cohomology.

Homología de intersección Comparación para perversidades diferentes.

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(a) the intersection homology groups $IH_{\star}^{\overline{p}}(A)$ are isomorphic for each loose perversity $\overline{p}$;

(b) each stratum S possess a tubular neighborhood U_S, whose homological monodromy is trivial, is a homology manifold. This generalizes a result of King, where the triviality of the U_S themselves was required.

Poincaré Duality of the basic intersection cohomology of a Killing foliation

We prove that the basic intersection cohomology $IH^{\star}_{\overline{p}}(M/\mathcal{F})$, where $\mathcal{F}$ is the singular foliation determined by an isometric action of a Lie group G on the compact manifold M, verifies the Poincar?é Duality Property.

Finitness of the basic intersection cohomology of a Killing foliation

We prove that the basic intersection cohomology $IH^{\star}_{\overline{p}}(M/\mathcal{F})$, where $\mathcal{F}$ is the singular foliation determined by an isometric action of a Lie group G on the compact manifold M, is finite dimensional.

Cohomological tautness for Riemannian foliations.

In this paper we present some new results on the tautness of Riemannian foliations in their historical context. The first part of the paper gives a short history of the problem. For a closed manifold, the tautness of a Riemannian foliation can be characterized cohomologically. We extend this cohomological characterization to a class of foliations which includes the foliated strata of any singular Riemannian foliation of a closed manifold.

Tautness for riemannian foliations on non-compact manifolds .

For a riemannian foliation $\mathcal{F}$ on a closed manifold M, it is known that $\mathcal{F}$ is taut (i.e. the leaves are minimal submanifolds) if and only if the (tautness) class defined by the mean curvature form κ_μ (relatively to a suitable riemannian metric μ) is zero. In the transversally orientable case, tautness is equivalent to the non-vanishing of the top basic cohomology group $H^n(M/\mathcal{F})$, where $n = \mathrm{codim}\mathcal{F}$. By the Poincar?é Duality, this last condition is equivalent to the non-vanishing of the basic twisted cohomology group $H^0_{\kappa_\mu}(M/\mathcal{F})$, when M is oriented. When M is not compact, the tautness class is not even defined in general. In this work, we recover the previous study and results for a particular case of riemannian foliations on non compact manifolds: the regular part of a singular riemannian foliation on a compact manifold (CERF).

Top dimensional group of the basic intersection cohomology for singular riemannian foliations.

It is known that, for a regular riemannian foliation on a compact manifold, the properties of its basic cohomology (non-vanishing of the top-dimensional group and Poincaré Duality) and the tautness of the foliation are closely related. If we consider singular riemannian foliations, there is little or no relation between these properties. We present an example of a singular isometric flow for which the top dimensional basic cohomology group is non-trivial, but its basic cohomology does not satisfy the Poincaré? Duality property. We recover this property in the basic intersection cohomology. It is not by chance that the top dimensional basic intersection cohomology groups of the example are isomorphic to either 0 or $\mathbb{R}$. We prove in this Note that this holds for any singular riemannian foliation of a compact connected manifold. As a Corollary, we get that the tautness of the regular stratum of the singular riemannian foliation can be detected by the basic intersection cohomology.

The BIC of a singular foliation defined by an abelian group of isometries.

We study the cohomology properties of the singular foliation $\mathcal{F}$ determined by an action $\Phi\colon\, G \times M \longrightarrow M$ where the abelian Lie group G preserves a riemannian metric on the compact manifold M. More precisely, we prove that the basic intersection cohomology $IH^{\star}_{\overline{p}}(M/\mathcal{F})$ is finite dimensional and verifies the Poincar?é Duality. This duality includes two well-known situations:

– Poincar?é Duality for basic cohomology (the action Φ is almost free).
– Poincar?é Duality for intersection cohomology (the group G is compact and connected).

The BIC of a conical fibration

In the paper we introduce the notions of a singular fibration and a singular Seifert fibration. These notions are natural generalizations of the notion of a locally trivial fibration to the category of stratified pseudomanifolds. For singular foliations defined by such fibrations we prove a de Rham type theorem for the basic intersection cohomology introduced the authors in a recent paper. One of important examples of such a structure is the natural projection onto the leaf space for the singular Riemannian foliation defined by an action of a compact Lie group on a compact smooth manifold.

Equivariant intersection cohomology of the circle actions

In this paper, we prove that the orbit space B and the Euler class of an action of the circle $\mathbb{S}^1$ on X determine both the equivariant intersection cohomology of the pseudomanifold X and its localization. We also construct a spectral sequence converging to the equivariant intersection cohomology of X whose third term is described in terms of the intersection cohomology of B.

The Gysin sequence for $\mathbb{S}^3$-actions on manifolds

We construct a Gysin sequence associated to any smooth $\mathbb{S}^3$-action on a smooth manifold.

A classical result says that a free action of the circle $\mathbb{S}^1$ on a topological space X is geometrically classified by the orbit space B and by a cohomological class $H^2(B, \mathbb{Z})$, the Euler class. When the action is not free we have a difficult open question: Π : "Is the space X determined by the orbit space B and the Euler class?" The main result of this work is a step towards the understanding of the above question in the category of unfolded pseudomanifolds. We prove that the orbit space B and the Euler class determine:

– the intersection cohomology of X,
– the real homotopy type of X.

Minimal Models for Non-Free Circle Actions.

Let $\Phi \colon \mathbb{S}^1 \times M \longrightarrow M$ be a smooth action of the unit circle $\mathbb{S}^1$ on a manifold M. In this work, we compute the minimal model of M in terms of the orbit space B and the fixed point set $F \subset B$, as a dg-module over the Sullivan's minimal model of B.

Cohomologie d’intersection des actions toriques simples.

We study the Leray-Serre spectral sequence associated to a simple action (one or two orbit type action) of the torus $\mathbb{T}$ on a manifold M. We describe the second term of this sequence in terms of the intersection cohomology of the orbit space $M/\mathbb{T}$.

A Gysin sequence for semifree actions of $\mathbb{S}^3$.

In this work we shall consider smooth semifree (i.e., free outside the fixed point set) actions of $\mathbb{S}^3$ on a manifold M. We exhibit a Gysin sequence relating the cohomology of M with the intersection cohomology of the orbit space $M/\mathbb{S}^3$. This generalizes the usual Gysin sequence associated with a free action of $\mathbb{S}^3$.

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Gysin sequences.

The main result of this work is the construction of the Gysin sequence

$$\cdots \longrightarrow H^{i}(M) \longrightarrow IH^{i-1}_{\overline{r}-\overline{2}}(M/\mathcal{F}) \xrightarrow{\wedge [e]} IH^{i+1}_{\overline{r}}(M/\mathcal{F}) \longrightarrow H^{i+1}(M) \longrightarrow \cdots ,$$

where $[e] \in IH^{2}_{\overline{2}}(M/\mathcal{F})$ is the Euler class. So, we can compute the deRham cohomology of M in terms of the basic intersection cohomology of $\mathcal{F}$. We end the work giving a geometrical interpretation of the vanishing of the Euler class in terms of the transversal triviality of $\mathcal{F}$.

The Euler class for flows of isometries.

We construct a Gysin sequence for a flow of isometries. We give a geometrical interpretation of the vanishing of the Euler class.

Euler y un balón de fútbol.

Champions League's logo is not a football ball.

A six dimensional compact symplectic solvmanifold without K?ähler structure.

We exhibit a non-K?hläerian compact symplectic solvmanifold. We prove that its minimal model is not formal.

Cosymplectic reduction for singular momentum maps

In this paper we prove that the cosymplectic reduction of cosymplectic manifolds with symmetry due to C. Albert may be obtained from the Marsden-Weinstein reduction theory. We also study the reduction of cosymplectic manifolds with singular momentum map by using the results of Sjamaar and Leman for the symplectic case.

Fuzzy filters.

In this paper a characterization of some fuzzy topological concepts, such as open sets, closed sets, adherent points, continuous functions,... is given by means of fuzzy filter convergence defined by the authors. F-ultrafilters are also characterized and relations between F-filters and F-nets are studied, getting results analogous to those for general topology.

Una nota sobre convergencia en espacios topológicos fuzzy.

We characterize some local fuzzy topological properties by using nets. We also study the filter convergence.

Hard Lefschetz property for isometric flows

The Hard Lefschetz Property (HLP) is an important property which has been studied in several categories of the symplectic world. For Sasakian manifolds, this duality is satisfied by the basic cohomology (so, it is a transverse property), but a new version of the HLP has been recently given in terms of duality of the cohomology of the manifold itself in [CMDNY15]. Both properties were proved to be equivalent (see [Lin16]) in the case of K-contact flows. In this paper we extend both versions of the HLP (transverse and not) to the more general category of isometric flows, and show that they are equivalent. We also give some explicit examples which illustrate the categories where the HLP could be considered.

The Gysin Braid for S^3-actions on manifolds

Given a smooth action of the sphere S^3 on a manifold M, we have previously constructed a Gysin sequence relating the cohomology of the manifold M and that of the orbit space M/S^3. This sequence involves an exotic term depending on the subset M^{S^1}. Notice that the orbit space is a stratified pseudomanifold, a kind of singular spaces where intersection cohomology applies. In the case where the the action is semi-free, the first author has already constructed a Gysin sequence relating the cohomology of M and the intersection cohomology of M/S^3. What happens if the action is not semi-free? This is the goal of this work. The situation is more complicated and we do not find a Gysin sequence but a Gysin braid relating the cohomology of M and the intersection cohomology of M/S^3. This braid also contains an exotic term depending this time on the intersection cohomology of M^{S^1}.

Hard Lefschetz property for S^3-actions

The Hard Lefschetz Property (HLP) has recently been formulated in the context of isometric flows without singularities on manifolds. In this category, two versions of the HLP (transverse and not) have been proven to be equivalent, thus generalizing what happens in the important cases of both K-contact and Sasakian manifolds. In this work we define both versions of the HLP for almost-free ^{3}-actions, and prove that they agree for actions satisfying a cohomological condition, which includes the important category of 3-Sasakian manifolds, where those two versions of the HLP are shown to be held. We also provide a family of examples of free actions of the 3-sphere which are not 3-Sasakian manifolds, but satisfy the HLP.

Starting with a manifold M and a semi-free action of S^3 on it, we have the Smith-Gysin sequence:

$$\cdots \longrightarrow H^{*}(M) \longrightarrow H^{*-3}\left(M/S^3, M^{S^3}\right) \oplus H^{*}\left(M^{S^3}\right) \longrightarrow H^{*+1}\left(M/S^3, M^{S^3}\right) \longrightarrow H^{*+1}(M) \longrightarrow \cdots,$$

In this paper, we construct a Smith-Gysin sequence that does not require the semi-free condition. This sequence includes a new term, referred to as the "exotic term," which depends on the subset M:

$$\cdots \longrightarrow H^{*}(M) \longrightarrow H^{*-3}\left(M/S^3, \Sigma/S^3\right) \oplus H^{*}\left(M^{S^3}\right) \oplus \left(H^{*-2}(M)\right)^{-\mathbb{Z}_2} \longrightarrow H^{*+1}\left(M/S^3, M^{S^3}\right) \longrightarrow H^{*+1}(M) \longrightarrow \cdots$$

Here, $\Sigma \subset M$ is the subset of points in M whose isotropy groups are infinite. The group $\mathbb{Z}_2$ acts on M by $j \in S^3$.