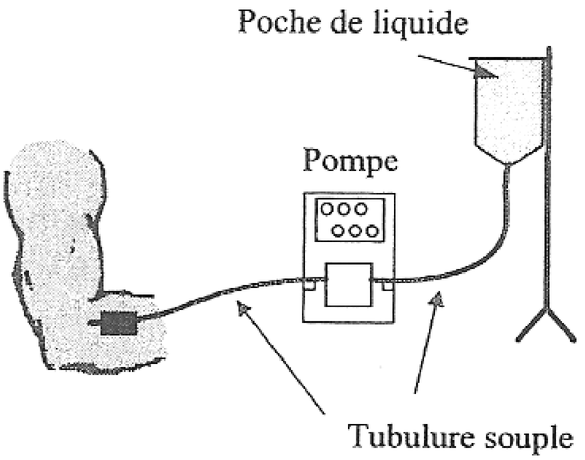
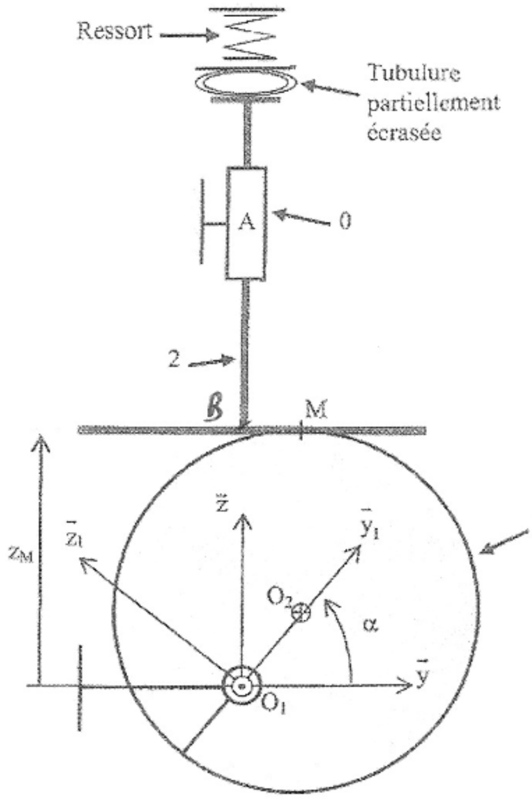
Corrigé TD Cinématique 4 : Contact entre 2 solides

Exercice 1 Pompe « Medicare »





$$\vec{V}(M \in 2/1) = \vec{V}(M \in 2/0) + \vec{V}(M \in 0/1)$$

$$\vec{V}(M \in 2/0) = \vec{V}(B \in 2/0) + \vec{\Omega}(2/0) \wedge \overrightarrow{BM} = \dot{z}.\vec{z}$$

$$\vec{V}(M \in 1/0) = \vec{V}(O_1 \in 1/0) + \vec{\Omega}(1/0) \wedge \overrightarrow{O_1M} = \vec{0} + \dot{\alpha}.\vec{x} \wedge (e.\vec{y}_1 + R.\vec{z})$$

$$\vec{V}(M \in 1/0) = e.\dot{\alpha}.\vec{z}_1 - R.\dot{\alpha}.\vec{y}$$

$$\vec{V}(M \in 2/1) = \dot{z}.\vec{z} - e.\dot{\alpha}.\vec{z}_1 + R.\dot{\alpha}.\vec{y}$$

$$\vec{z}_1 = -\sin\alpha.\vec{y} + \cos\alpha.\vec{z}$$

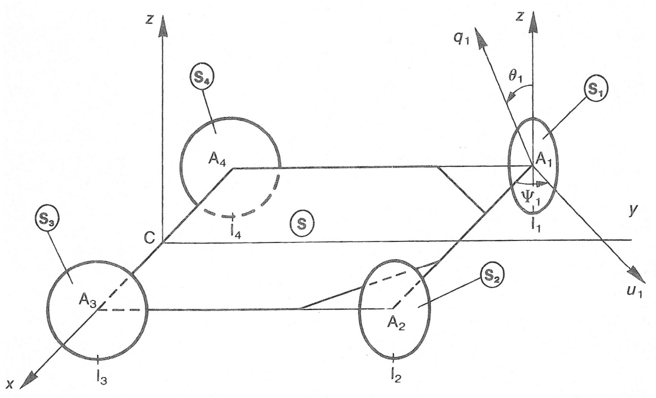
$$\vec{V}(M \in 2/1) = (e.\dot{\alpha}.\sin\alpha + R.\dot{\alpha}).\vec{y} + (\dot{z} - e.\dot{\alpha}.\cos\alpha).\vec{z}$$

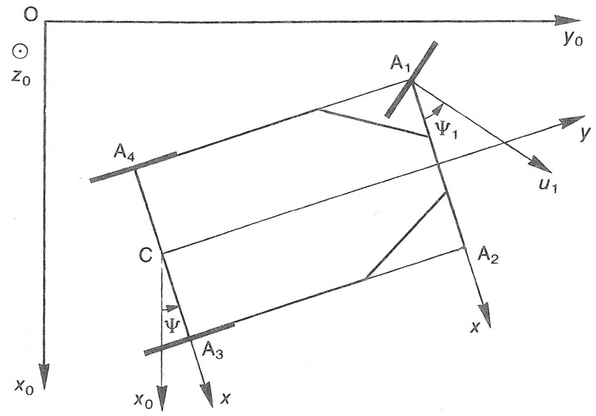
La vitesse de glissement est dans la direction $\vec{y}$ ⇨ $\dot{z} = e.\dot{\alpha}.\cos\alpha$

$$\{V1/0\} = \begin{Bmatrix} \vec{\Omega}(1/0) \\ \vec{V}(O \in 1/0) \end{Bmatrix}_O = \begin{Bmatrix} \dot{\alpha}.\vec{x} \\ \vec{0} \end{Bmatrix}_O \qquad \{V2/0\} = \begin{Bmatrix} \vec{\Omega}(2/0) \\ \vec{V}(B \in 2/0) \end{Bmatrix}_B = \begin{Bmatrix} \vec{0} \\ \dot{z}.\vec{z} \end{Bmatrix}_B$$

$$\{V2/1\} = \begin{Bmatrix} \vec{\Omega}(2/1) \\ \vec{V}(M \in 2/1) \end{Bmatrix}_M = \begin{Bmatrix} -\dot{\alpha}.\vec{x} \\ -\dot{\lambda}.\vec{y} \end{Bmatrix}_M$$

Exercice 2 Véhicule à 4 roues





1. I est l'intersection des droites $(A_1,\vec{u}_1)$ et $(C,\vec{x})$

2. La roue (2) est perpendiculaire à (IA_2) .

3. $\tan\psi_1 = \dfrac{l}{\rho - d}$ $\tan\psi_2 = \dfrac{l}{\rho + d}$

4. $\vec{V}(C \in S/R_0) = \vec{V}(I \in S/R_0) + \vec{\Omega}(S/R_0) \wedge \overrightarrow{IC}$

$V.\vec{y} = 0 + \dot{\psi}.\vec{z}_0 \wedge \rho.\vec{x} = \dot{\psi}.\rho.\vec{y}$ $V = \dot{\psi}.\rho$

5. Roulement sans glissement en I_1 ⇨ $\vec{V}(I_1 \in 1/R_0) = \vec{0}$

$$\vec{V}(I_1 \in 1/R_0) = \vec{V}(I_1 \in 1/S) + \vec{V}(I_1 \in S/R_0) = \vec{0}$$

$$\vec{V}(I_1 \in 1/S) = \vec{V}(A_1 \in 1/S) + \vec{\Omega}(1/S) \wedge \overrightarrow{A_1I_1} = \dot{\theta}_1.\vec{u}_1 \wedge (-r.\vec{z}) = r.\dot{\theta}_1.\vec{v}_1$$

$$\vec{V}(I_1 \in S/R_0) = \vec{V}(I \in S/R_0) + \vec{\Omega}(S/R_0) \wedge \overrightarrow{II_1}$$

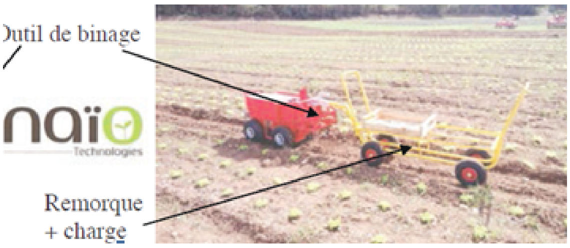
$$\vec{V}(I_1 \in S/R_0) = \dot{\psi}.\vec{z} \wedge .\vec{u}_1 \wedge (R_1.\vec{u}_1 - r.\vec{z}) = R_1.\dot{\psi}.\vec{v}_1$$

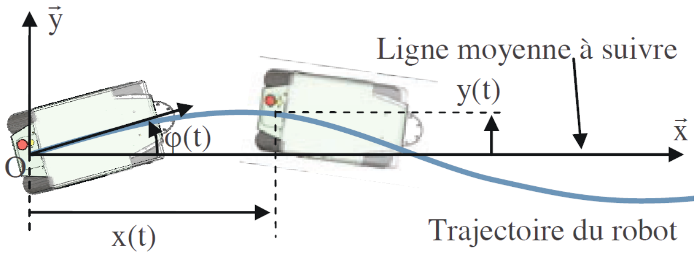
$\dot{\theta}_1.r = R_1.\dot{\psi}$ $\dot{\theta}_1 = \dfrac{R_1.\dot{\psi}}{r}$

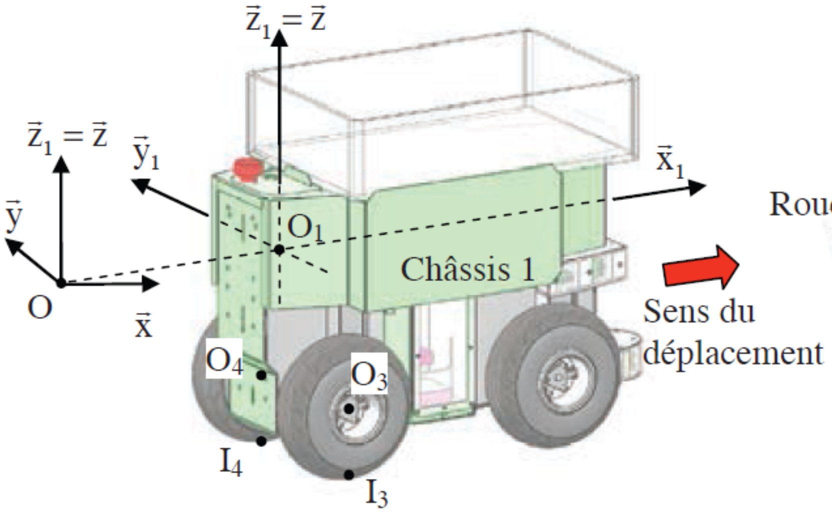
6. *De même* $\dot{\theta}_2 = \dfrac{R_2.\dot{\psi}}{r}$

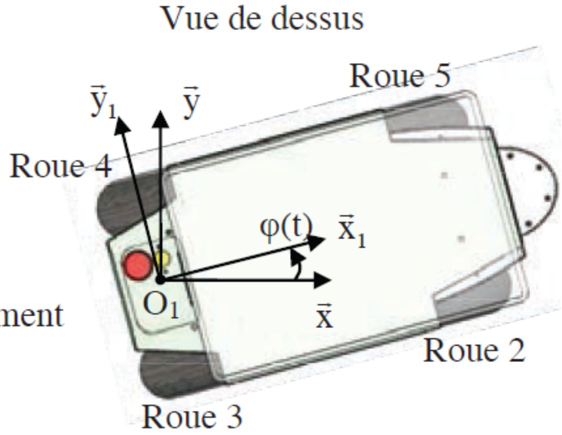
⇨ *en virage, les roues ne tournent pas à la même vitesse !!!*

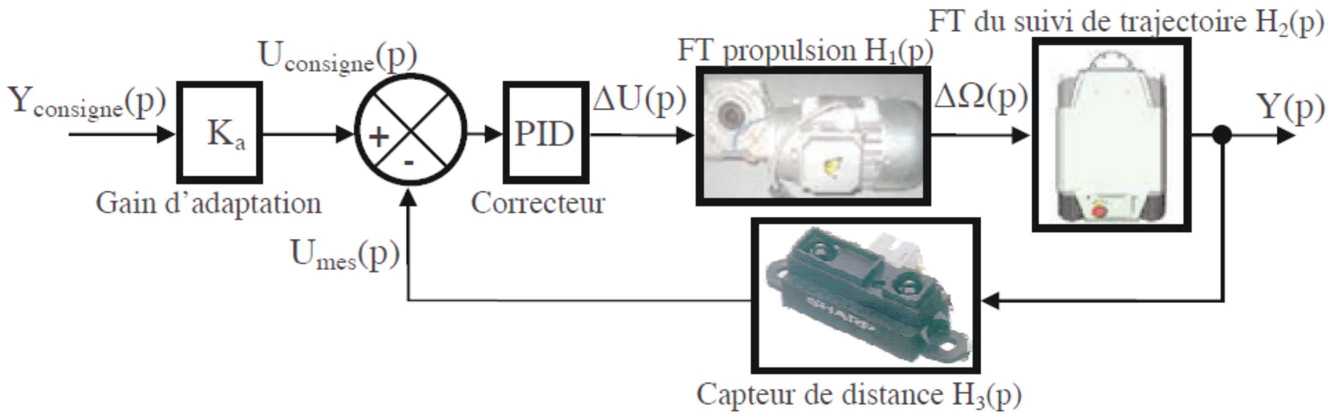
Exercice 3 Robot de maraichage (CCP MP 2016)











Question 1

En I_3 la condition de roulement sans glissement entre la roue 3 et le sol s'écrit :

$$\vec{V}\left(I_3 \in roue3/sol\right) = \vec{0} \ et \ \vec{\Omega}(roue3/sol).\vec{y}_3 \neq 0$$

Que l'on peut aussi écrire : $\vec{V}\left(I_3 \in 3/0\right) = \vec{0} \ et \ \vec{\Omega}(roue3/0).\vec{y}_3 \neq 0$

$$\vec{V}\left(I_3 \in 3/0\right) = \vec{0} = \vec{V}\left(I_3 \in 3/1\right) + \vec{V}\left(I_3 \in 1/0\right)$$

$$\vec{V}\left(I_3 \in 3/0\right) = \vec{0} = \vec{V}\left(0_3 \in 3/1\right) + \vec{\Omega}(3/1) \wedge \overrightarrow{0_3 I_3} + \vec{V}\left(0_1 \in 1/0\right) + \vec{\Omega}(1/0) \wedge \overrightarrow{0_1 I_3}$$

$$\vec{0} = \vec{0} + \omega_d \vec{y}_1 \wedge (-r\vec{z}) + V\vec{x}_1 + \dot{\varphi}\vec{z} + (-e\vec{y}_1 - h\vec{z} - r\vec{z})$$

$$-r\omega_d \vec{x}_1 + V\vec{x}_1 + e\dot{\varphi}\vec{x}_1 = \vec{0}$$

En projection sur $\vec{x}_1$: $\quad -r\omega_d + V + e\dot{\varphi} = 0$

Question 1

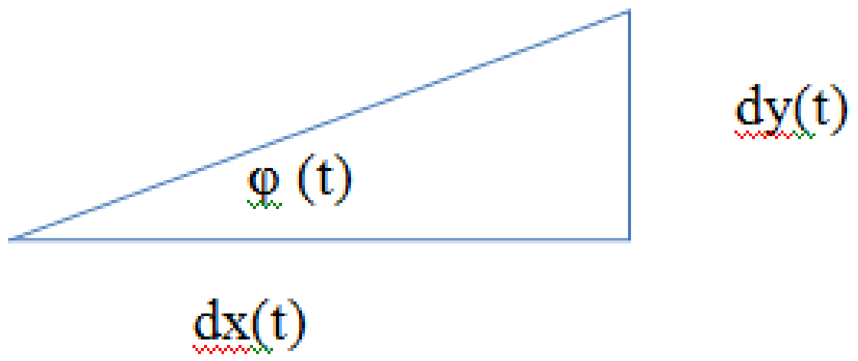
De même : $-r\omega_d + V - e\dot{\varphi} = 0$

Question 2 On a : $\omega_d = \dfrac{V + e\dot{\varphi}}{r}$ et $\omega_g = \dfrac{V - e\dot{\varphi}}{r}$

On en déduit : $\Delta\omega = \omega_d - \omega_g = \dfrac{V + e\dot{\varphi}}{r} - \dfrac{V - e\dot{\varphi}}{r}$ $\Delta\omega = \dfrac{2e}{r}\dot{\varphi} = \dfrac{2e}{r}\dfrac{d\varphi}{dt}$

$H_{21}(p) = \dfrac{\Phi(p)}{\Delta\Omega(p)} = \dfrac{r}{2ep}$ (Condition initiale nulle)

Question 3



Question 4

$$\tan\varphi(t) = \frac{dy(t)}{dx(t)} \; si\,\varphi\, est\ petit\ \tan\varphi(t) = \varphi(t) \quad d'où\ \varphi(t) = \frac{dy(t)}{dx(t)} = \frac{dy(t)}{dt}.\frac{dt}{dx(t)}$$

$$\frac{dy(t)}{dt} = \varphi(t)\frac{dx(t)}{dt} \qquad \dot{y}(t) = \varphi(t)\dot{x}(t) = \varphi(t)V$$

On avait : $H_{22}(p) = \dfrac{Y(p)}{\Phi(p)} = \dfrac{V}{p}$ (condition initiale nulle)

On en déduit :

$$H_2(p) = H_{21}(p)H_{22}(p) = \frac{r}{2ep}\frac{V}{p} = \frac{rV}{2ep^2}$$