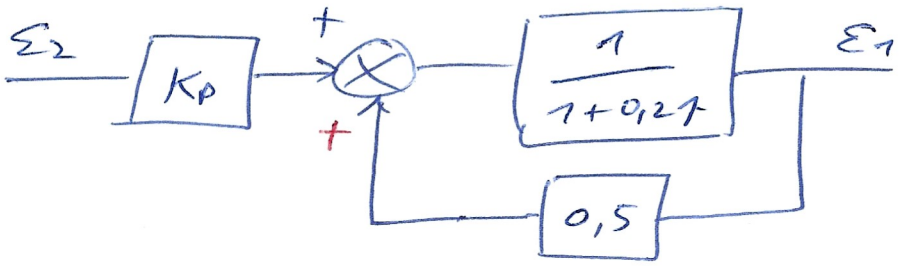
Correction du DM de SI, MPSI 1, PCSI 1, nov 21

(Stabilisateur actif, Mines 18)

Q1



$$H_1 = \frac{\varepsilon_1}{\varepsilon_2} = K_p \cdot \frac{1}{1+0,2\tau-0,5} = \frac{K_p}{0,5+0,2\tau} = \frac{10.K_p}{5+2\tau}$$

$$H_2(\tau) = \frac{10.K_p}{\tau(5+2\tau)+10.K_p} = \frac{1}{\frac{1}{5K_p}\tau^2 + \frac{1}{2K_p}\tau + 1}$$

$$H_2(\tau) = \frac{K}{\frac{\tau^2}{\omega_n^2} + \frac{2z}{\omega_n}\tau + 1} \Longrightarrow \left| \begin{array}{l} K = 1 \\ \omega_n = \sqrt{5.K_p} \end{array} \right.$$

$$\frac{2z}{\omega_n} = \frac{1}{2K_p} \Longrightarrow z = \frac{\sqrt{5}}{5\sqrt{K_p}}$$

On veut $z = 1$

$$\Longrightarrow K_p = \frac{5}{16} = 0,31$$

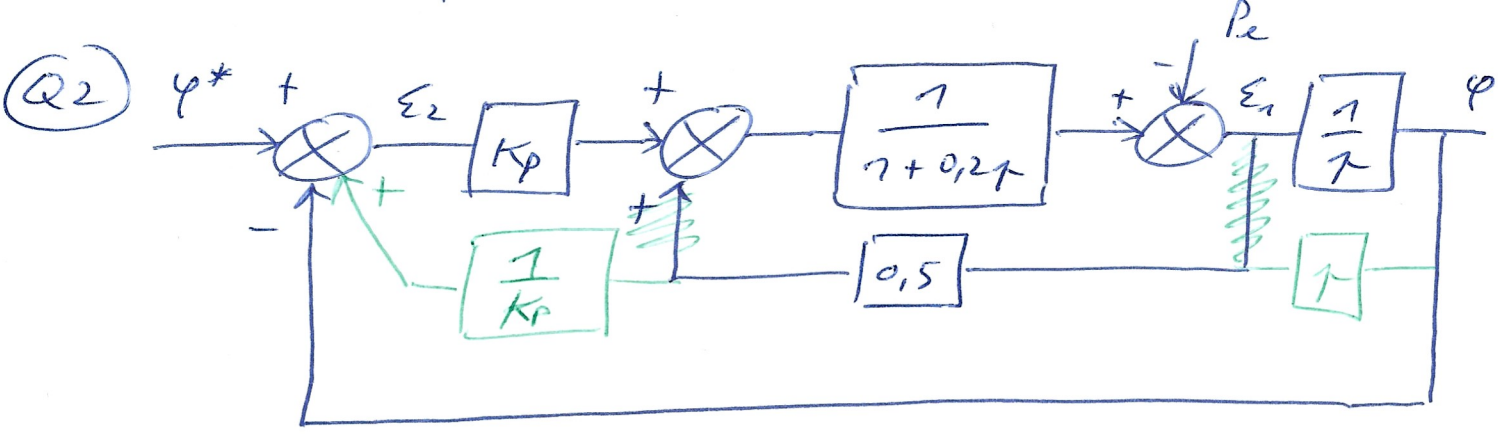
Performances :

$K = 1 \Longrightarrow$ Précis

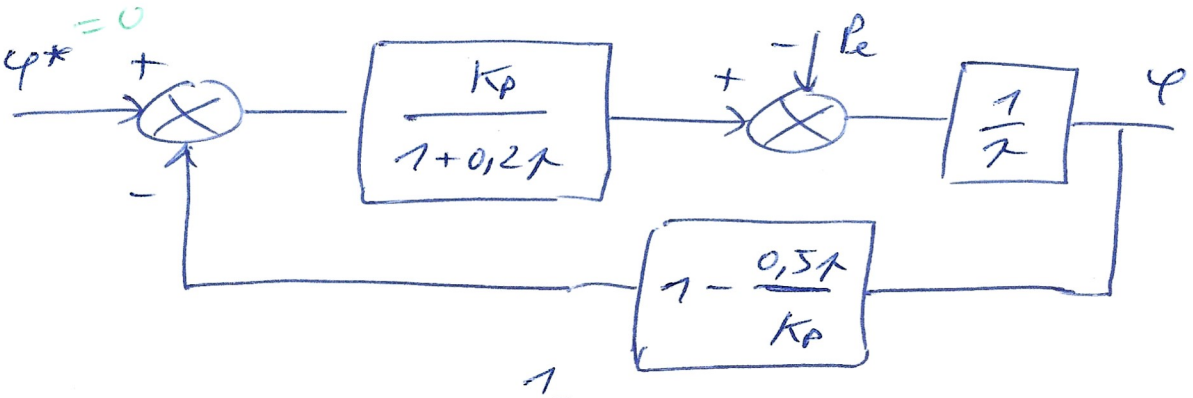
$t_{5\%} = \frac{5}{\omega_n} = \cancel{1,12}\ \cancel{s}$. (rapidité)

$z = 1 \Longrightarrow$ Pas de dépassement

Q2



(2)



$$H_3(\tau) = \frac{\varphi}{P_e} = \frac{\frac{1}{\tau}}{1 + \frac{1}{\tau}\,\frac{K_p - 0{,}5\tau}{K_p}\,\frac{K_p}{1+0{,}2\tau}}$$

$$H_3(\tau) = \frac{1+0{,}2\tau}{\tau(1+0{,}2\tau) + K_p - 0{,}5\tau} = \frac{1+0{,}2\tau}{0{,}2\tau^2 + 0{,}5\tau + K_p}$$

Entrée $P_e(\tau) = \frac{1}{\tau}$ $\qquad \varphi(\tau) = H_3(\tau)\, P_e(\tau)$.

$$\lim_{t\to\infty} \varphi(t) = \lim_{\tau\to 0} \tau\,\varphi(\tau) = \frac{1}{K_p} \neq 0$$

Avec $K_p = \frac{5}{16}$ on a $\lim_{t\to\infty} \varphi(t) = \frac{16}{5}$ rad.

Conclusion : Comportement en poursuite (réponse à une consigne) et en régulation (réponse à une perturbation.

Illustration :