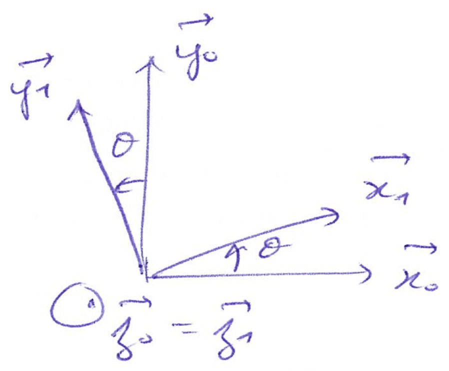
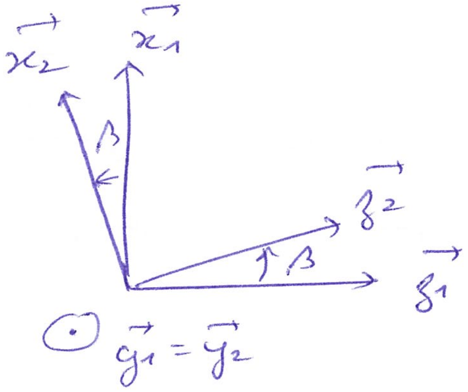
$\boxed{\text{Exo 1}}$ $\boxed{\text{DS MPSI 1, SI, mars 22.}}$

Q1





Q2 $\vec{v}(G_3 \in 2/0) = \left(\dfrac{d\vec{OG_3}}{dt}\right)_0$

$$\vec{OG_3} = a\vec{x_1} + b\vec{x_2}$$

$$\left(\frac{d\vec{x_2}}{dt}\right)_0 = \left(\frac{d\vec{x_2}}{dt}\right)_1 + \vec{\Omega}_{1/0} \wedge \vec{x_2}$$

$$= -\dot{\beta}\vec{z_2} + \dot{\theta}\vec{z_1} \wedge (\cos\beta\,\vec{x_1} - \sin\beta\,\vec{z_1})$$

$$= -\dot{\beta}\vec{z_2} + \dot{\theta}\cos\beta\,\vec{y_1} \qquad (\times b)$$

$$\vec{v}(G_3 \in 2/0) = a\dot{\theta}\vec{y_1} + b\dot{\theta}\cos\beta\,\vec{y_1} - b\dot{\beta}\vec{z_2}$$

$$= \underbrace{(a + b\cos\theta)\dot{\theta}\vec{y_1}}_{\substack{\vec{v}(G_3 \in 1/0) \\ \text{entrainement}}} \underbrace{- b\dot{\beta}\vec{z_2}}_{\substack{\vec{v}(G_3 \in 2/1) \\ \text{relative.}}}$$

Q3 $\vec{v}(G_3 \in 2/0) = \underbrace{\vec{v}(A \in 2/0)} + \vec{\Omega}_{2/0} \wedge \vec{AG_3}$

$$\underbrace{\vec{v}(A \in 2/1)}_{\vec{0}} + \underbrace{\vec{v}(A \in 1/0)}_{\vec{v}(O \in 1/0) + \vec{\Omega}_{1/0} \wedge \vec{OA}}$$

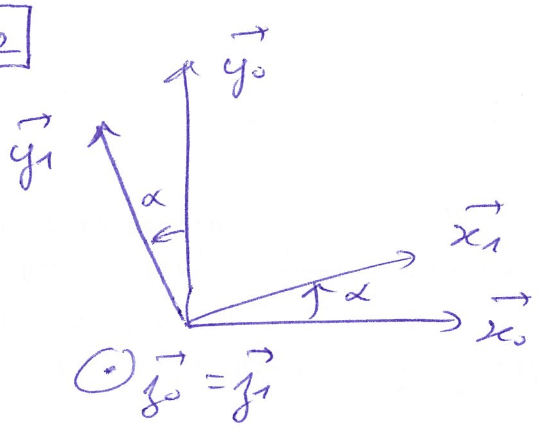
Finalement :

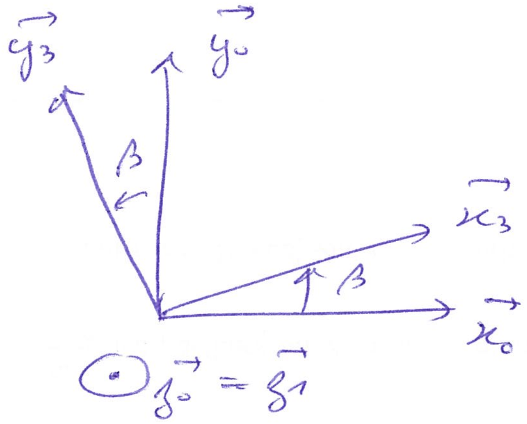
$$\vec{v}(G_3 \in 2/0) = \vec{\Omega}_{2/0} \wedge \vec{AG_3} + \vec{\Omega}_{1/0} \wedge \vec{OA}$$

(2)

$\boxed{\text{Exo 2}}$

Q1





Q2 $\vec{OC} + \vec{CB} = \vec{OB}$

$-a\vec{x_0} + b\vec{x_3} = \lambda \vec{x_1}$

$$\left| \begin{array}{ll} \vec{x_3} = \cos\beta\, \vec{x_0} + \sin\beta\, \vec{y_0} & (\times b) \\ \vec{x_1} = \cos\alpha\, \vec{x_0} + \sin\alpha\, \vec{y_0} & (\times \lambda) \end{array} \right.$$

$$\left| \begin{array}{l} -a + b\cos\beta = \lambda \cos\alpha \\ b\sin\beta = \lambda \sin\alpha \end{array} \right. \Longrightarrow$$

$$(-a + b\cos\beta)^2 + (b\sin\beta)^2 = \lambda^2$$

$$\lambda = \sqrt{b^2 + a^2 - 2ab\cos\beta}$$

3

$\boxed{\text{Exo 3}}$ Hublex

Q2 $V = r_c\, \omega_{10}$

Q3 Rsg $\Rightarrow$ $\vec{V}(I \in 2/0) = \vec{0} = \vec{V}(I \in 2/1) + \vec{V}(I \in 1/0)$

$$\vec{V}(I \in 2/1) = \vec{V}(A \in 2/1) + \vec{\Omega}_{2/1} \wedge \overrightarrow{AI}$$
$$= \vec{0} + \omega_{21}\, \vec{x_1} \wedge (-R\, \vec{z_1}) = R.\omega_{21}.\vec{y_1}$$

$$\vec{V}(I \in 1/0) = \vec{V}(O_1 \in 1/0) + \vec{\Omega}_{1/0} \wedge \overrightarrow{O_1 I}$$
$$= V\vec{y_1} + \omega_{10}\, \vec{z_1} \wedge \left(-\frac{L}{2}\vec{x_1} + \cdots \vec{z_0}\right)$$
$$= \left(V - \omega_{10}\frac{L}{2}\right)\vec{y_1}$$

Rsg $\Rightarrow$ $R\omega_{21} + V - \omega_{10}\frac{L}{2} = 0$

$$\omega_{21} = -\frac{1}{R}\left(V - \frac{L}{2}\omega_{10}\right)$$

Q4 $$\omega_{41} = -\frac{1}{Rk}\left(V - \frac{L}{2}\omega_{10}\right)$$

Q5 De même, $\omega_{51} = -\frac{1}{Rk}\left(V + \frac{L}{2}\omega_{10}\right)$