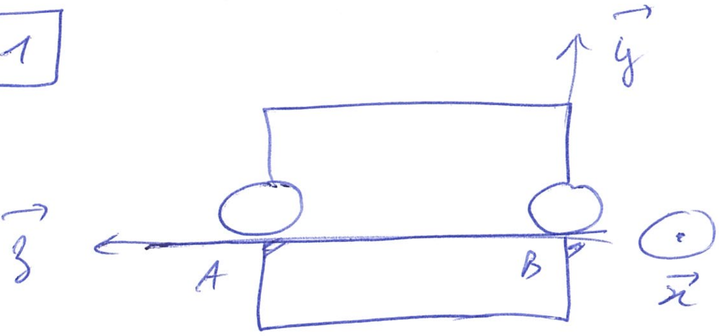
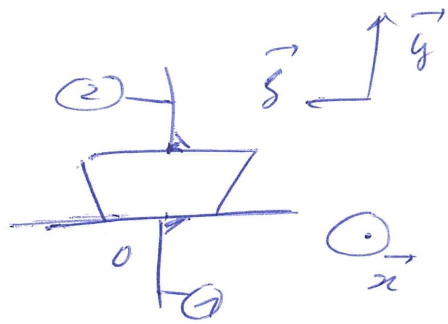
DS MPSI, mars 23

Exo 1



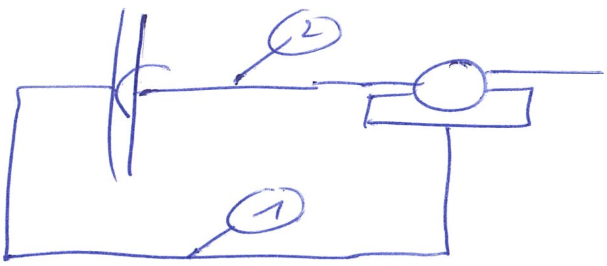


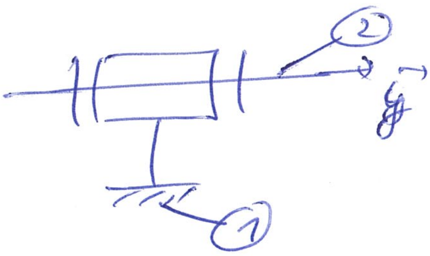
$$\{\mathcal{V}^{2/1}\} = \begin{Bmatrix} 0 & u \\ \beta & 0 \\ \gamma & w \end{Bmatrix}_O$$

Linéaire annulaire

~~Sphè~~ cylindre plan.

Exo 2

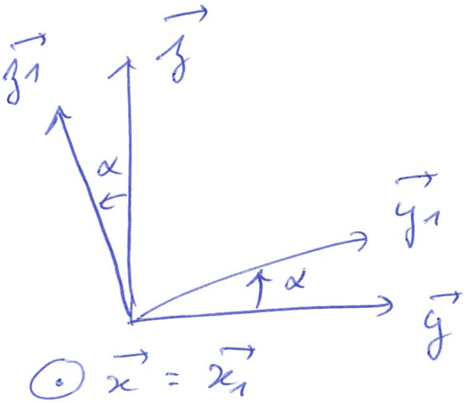


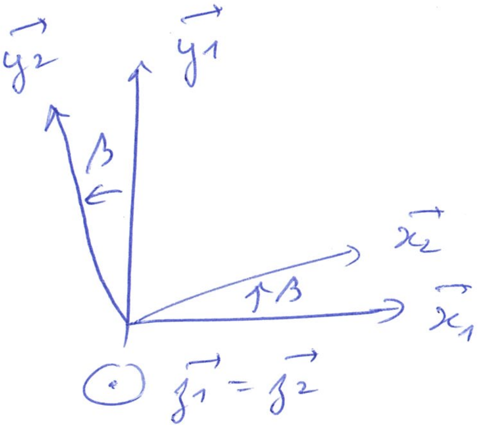


$$\{\overleftrightarrow{\mathcal{V}}\} = \begin{Bmatrix} 0 & 0 \\ 0 & v \\ 0 & 0 \end{Bmatrix}_A$$

$\boxed{\text{Exo 3}}$

Q1





Q2 $\{\mathcal{V}\,2/0\} = \left\{ \begin{matrix} \vec{\Omega}\,2/0 \\ \vec{v}(B \in 2/0) \end{matrix} \right\}_B$

$\vec{\Omega}\,2/0 = \dot{\alpha}\,\vec{x_1} + \dot{\beta}\,\vec{z_1}$

$\vec{v}(B\in 2/0) = \left(\dfrac{d\,\vec{OB}}{dt}\right)_0 \qquad \vec{OB} = \vec{OA} + \vec{AB} = r\,\vec{y_1} + l\,\vec{x_2}$

$$\left(\frac{d\vec{x_2}}{dt}\right)_0 = \left(\frac{d\vec{x_2}}{dt}\right)_1 + \vec{\Omega}\,1/0 \wedge \vec{x_2}$$

$$= \dot{\beta}\,\vec{y_2} + \dot{\alpha}\,\vec{x_1} \wedge (\cos\beta\,\vec{x_1} + \sin\beta\,\vec{y_1})$$

$$= \dot{\beta}\,\vec{y_2} + \dot{\alpha}\sin\beta\,\vec{z_1} \qquad (\times l)$$

$$\vec{v}(B\in 2/0) = r\dot{\alpha}\,\vec{y_1} + l\dot{\alpha}\sin\beta\,\vec{z_1} + l\dot{\beta}\,\vec{y_2}$$

$$= l\dot{\beta}\,\vec{y_2} + (r + l\sin\beta)\,\dot{\alpha}\,\vec{z_1}$$

$$= \vec{v}(B\in 2/1) + \vec{v}(B \in 1/0)$$

Q3 $\vec{AC} = \vec{AB} + \vec{AC}$

$x\,\vec{x} = l\,\vec{x_2} + l\,\vec{x_3}$ $\Big\} \longrightarrow x = 2\,l\cos\beta$

③ $\boxed{\text{Exo 4}}$ (Q1) y varie de 0 à $(R_4 - a)$

(Q2) $\vec{v}(B \in 3/2) = \vec{0} = \vec{v}(B \in 3/1) + \vec{v}(B \in 1/2)$

$\vec{v}(B \in 2/1) = \vec{v}(A \in 2/1) + \vec{\Omega}_{2/1} \wedge \vec{AB}$

$= \vec{0} + \omega_2 \vec{x} \wedge (\dots \vec{x} + y \vec{y})$

$= y \, \omega_2 \vec{z}$

$\vec{v}(B \in 3/1) = \vec{v}(E \in 3/1) + \vec{\Omega}_{3/1} \wedge \vec{EB}$

$= \vec{0} + \omega_3 \vec{y} \wedge (-r \vec{x})$

$= r \, \omega_3 \vec{z}$

$\Rightarrow \; y \, \omega_2 = r \, \omega_3 \; \Rightarrow \; \boxed{\omega_3 = \frac{y}{r} \cdot \omega_2}$

(Q3) $\vec{v}(C \in 4/3) = \vec{0} = \vec{v}(C \in 4/1) + \vec{v}(C \in 1/3)$

$\vec{v}(C \in 4/1) = \vec{v}(D \in 4/1) + \vec{\Omega}_{4/1} \wedge \vec{DC}$

$= \vec{0} + \omega_4 \vec{x} \wedge [\dots \vec{x} + (a+y) \vec{y}]$

$= (a+y) \, \omega_4 \vec{z}$

$\vec{v}(C \in 3/1) = \vec{v}(E \in 3/1) + \vec{\Omega}_{3/1} \wedge \vec{EC}$

$= \vec{0} + \omega_3 \vec{y} \wedge r \vec{x}$

$= -r \, \omega_3 \vec{z}$

$\Rightarrow \; (a+y) \, \omega_4 = -r \, \omega_3 \; \Rightarrow \; \boxed{\omega_4 = -\frac{r}{a+y} \omega_3}$

(Q4) $\boxed{\omega_4 = -\frac{y}{a+y} \cdot \omega_2}$