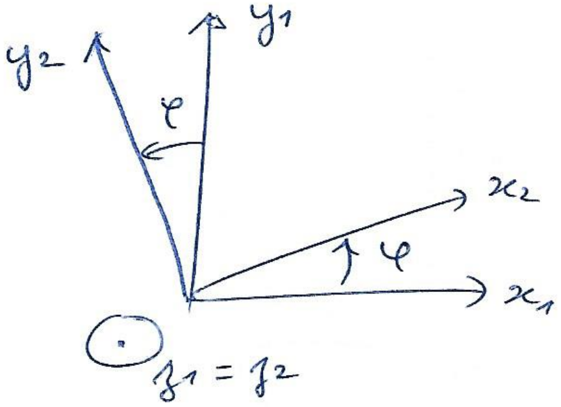
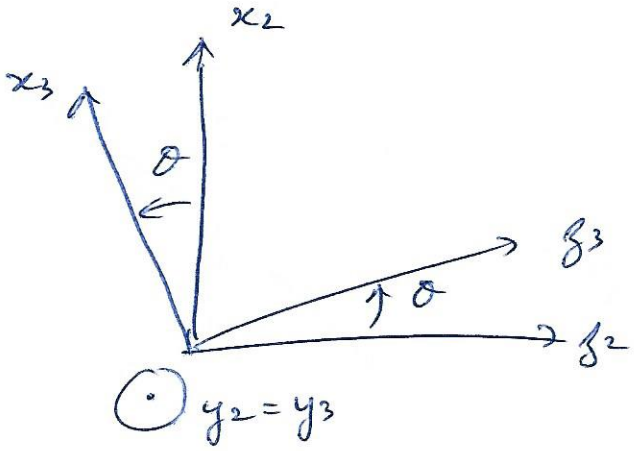
1

Manège

Q1 $\vec{R_c}(3/1) = m_3 . \vec{v}(G_3 \in 3/1)$

$\vec{OG_3} = a\,\vec{x_2} - \ell.\vec{z_3}$





$$\left(\frac{d\vec{z_3}}{dt}\right)_1 = \left(\frac{d\vec{z_3}}{dt}\right)_2 + \underbrace{\vec{\Omega}_{2/1}}_{\dot\varphi\vec{z_2}} \wedge \vec{z_3} = \dot\theta\,\vec{x_3} + \dot\varphi\sin\theta\,\vec{y_3} \quad (\times -\ell)$$

$$\vec{v}(G_3 \in 3/1) = a\dot\varphi\,\vec{y_2} - \ell\dot\theta\,\vec{x_3} - \ell\dot\varphi\sin\theta\,\vec{y_3}$$

$$= -\ell\dot\theta\,\vec{x_3} + (a - \ell\sin\theta)\dot\varphi\,\vec{y_3}$$

$$\vec{\sigma}(A, 3/1) = \overline{\overline{I_A}}(3).\vec{\Omega}_{3/1} + m_3\,\vec{AG_3} \wedge \vec{v}(A \in 3/1)$$

$\vec{\Omega}_{3/1} = \dot\varphi\,\vec{z_2} + \dot\theta\,\vec{y_3}$ $\qquad \vec{z_2} = -\sin\theta\,\vec{x_3} + \cos\theta\,\vec{z_3}$

$\vec{AG_3} = -\ell\,\vec{z_3}$ $\qquad \vec{v}(A \in 3/1) = a\dot\varphi\,\vec{y_3}$.

$$\vec{\sigma}(A, 3/1) = \begin{bmatrix} A & & \\ & B & \\ & & 0 \end{bmatrix} \begin{bmatrix} -\dot\varphi\sin\theta \\ \dot\theta \\ \dot\varphi\cos\theta \end{bmatrix} + m_3\,a\,\ell\,\dot\varphi\,\vec{x_3}$$

$$\vec{\sigma}(A, 3/1) = (m_3\,a\,\ell - A\sin\theta)\dot\varphi\,\vec{x_3} + B\dot\theta\,\vec{y_3}$$

Manège (suite).

Q2 $\vec{\delta}(A, 3/1) = \left(\dfrac{d\vec{\sigma}(A, 3/1)}{dt}\right) + m_3 \underbrace{\vec{v}(A/1) \wedge \vec{v}(G_3 \in 3/1)}_{a b \dot{\varphi}\dot{\theta} \vec{z_3}}$

$\vec{v}(A/1) = a\dot{\varphi}\vec{y_3}$

$\vec{v}(G_3 \in 3/1) = -b\dot{\theta}\vec{x_3} + (a - b\sin\theta)\dot{\varphi}\vec{y_3}$

<u>Rem</u> : On cherche $\vec{\delta}(A, 3/1).\vec{y_3}$ $\quad (\vec{y_3} = \vec{y_2})$.

$$\left(\frac{d\vec{\sigma}(A,3/1)}{dt}\right).\vec{y_3} = \left(\frac{d\vec{\sigma}(A,3/1).\vec{y_3}}{dt}\right)_1 - \vec{\sigma}(A,3/1).\underbrace{\left(\frac{d\vec{y_2}}{dt}\right)_1}_{-\dot{\varphi}\vec{x_2}}$$

$$\vec{\delta}(A,3/1).\vec{y_3} = B\ddot{\theta} + (m_3 a b - A\sin\theta)\dot{\varphi}^2\cos\theta$$

Q3 $\{\mathcal{T}_{2\to 3}\} = \begin{Bmatrix} X & L \\ Y & 0 \\ Z & N \end{Bmatrix}_{A, B_2}$

$\vec{M}_{pes}(A) = \vec{M}_{pes}(G_3) + \vec{AG_3} \wedge \vec{P}$

$= \vec{0} + (-b\vec{z_3}) \wedge (-m_3 g \vec{z_2}) = -m_3 g b \sin\theta \, \vec{y_2}$

<u>On isole (3), TMD sur $(A, \vec{y_3})$</u>

$$B\ddot{\theta} + (m_3 a b - A\sin\theta)\dot{\varphi}^2\cos\theta = -m_3 g b \sin\theta.$$

Q4 $\dot{\varphi} = cte$; $\theta = cte$, $\ddot{\theta} = 0$, de plus $A\sin\theta \ll m_3 a b$

$\Longrightarrow \quad m_3 a b \dot{\varphi}^2 \cos\theta = -m_3 g b \sin\theta.$

$$\tan\theta = -\frac{a.\dot{\varphi}^2}{g}$$