$\boxed{\text{Correction DS PSI, MP, octobre 22}}$

<u>Stabilisation, Centrale PSI 22</u>

(Q1) Direction droite-verticale : $\left|\begin{array}{l} u_d^{max} = 3,5 \text{ mm} \\ u_d^{min} = -2,3 \text{ mm} \end{array}\right.$ plus ou moins $\pm$ 0,2 mm

Plan frontal $\left|\begin{array}{l} u_f^{max} = 0,2 \text{ mm} \\ u_f^{min} = -0,3 \text{ mm} \end{array}\right.$

(Q2) $\overrightarrow{P_0P} = \overrightarrow{P_0O_0} + \overrightarrow{O_0P} = -L\vec{z_0} + L\vec{z_1} =$

$\vec{z_1} = \cos\theta_y\, \vec{z_1}' + \sin\theta_y\, \vec{x_1}'$

$\vec{z_1}' = -\sin\theta_x\, \vec{y_0} + \cos\theta_x\, \vec{z_0}$; $\vec{x_1}' = \vec{x_0}$

$\vec{z_1} = \sin\theta_y\, \vec{x_0} - \cos\theta_y \sin\theta_x\, \vec{y_0} + \cos\theta_y \cos\theta_x\, \vec{z_0}$ $(\times L)$

$\overrightarrow{P_0P} = L\left[\sin\theta_y\, \vec{x_0} - \cos\theta_y \sin\theta_x\, \vec{y_0} + (\cos\theta_y\cos\theta_x - 1)\vec{z_0}\right]$

$u_d = -L\cos\theta_y \sin\theta_x$

$u_f = L\sqrt{(\sin\theta_y)^2 + (\cos\theta_y\cos\theta_x - 1)^2}$

(Q3) $u_d \simeq -L\,\theta_x$; $u_f \simeq L.\theta_y$

$\Delta\theta_x = \dfrac{u_d^{min} - u_d^{max}}{L} = \dfrac{-2,3 - 3,5}{0,3.10^{+3}} = -0,019 \text{ rad}$

$\Delta\theta_y = \dfrac{u_f^{max} - u_f^{min}}{L} = \dfrac{0,2 + 0,3}{0,3.10^{3}} = 0,0017 \text{ rad}.$

$\Delta\theta_x \gg \Delta\theta_y$ o

on peut négliger la rotation autour de $(O, \vec{y_1})$

$\Rightarrow$ on peut considérer la liaison sphérique en O

comme une pivot d'axe $(O_0, \vec{x_0})$

(Q5) Compensation sur $(O_0, \vec{x_0})$.

Q5 $\vec{\sigma}(G_3, 3/0) = \begin{bmatrix} A & & \\ & B & \\ & & A \end{bmatrix} \begin{bmatrix} 0 \\ \dot{\theta}_3 \\ \dot{\theta}_2 \end{bmatrix} = B\dot{\theta}_3 \vec{y_2} + A\dot{\theta}_2 \vec{z_2}$

Q6 $\vec{\delta}(G_3, 3/0) = -B\dot{\theta}_3\dot{\theta}_2 \vec{x_2} + B\ddot{\theta}_3 \vec{y_2} + A\ddot{\theta}_2 \underset{\vec{z_1}}{\underset{\shortparallel}{\vec{z_2}}}$

Q7 <u>On isole (1+2+3), TMD en G_3, dans B_1</u> ⚠

$$\left| \begin{array}{l} \vec{x_2} = \cos\theta_2 \vec{x_1} + \sin\theta_2 \vec{y_1} \\ \vec{y_2} = -\sin\theta_2 \vec{x_1} + \cos\theta_2 \vec{y_1} \end{array} \right.$$

$$\left| \begin{array}{l} L_{01} = -B\dot{\theta}_3\dot{\theta}_2 \cos\theta_2 - B\ddot{\theta}_3 \sin\theta_2 \\ M_{01} = -B\dot{\theta}_3\dot{\theta}_2 \sin\theta_2 + B\ddot{\theta}_3 \cos\theta_2 \\ N_{01} = A\ddot{\theta}_2 \end{array} \right.$$

Q8 $\omega_3 = cte \Longrightarrow \ddot{\theta}_3 = 0$

$\vec{M}(G_3) = \vec{M}(P) + \vec{G_3P} \wedge f_c \vec{y_1} =$

$= \vec{0} + (L\vec{z_1} - L_{G_3}\vec{z_1} + H_{G_3}\vec{y_1}) \wedge f_c \vec{y_1}$

$= (L_{G_3} - L) f_c \vec{x_1}$

$L_{01} + (L_{G_3} - L) f_c = -B\omega_3 \dot{\theta}_2 \cos\theta_2$

$L_{01} = 0 \Longrightarrow \dot{\theta}_2 = \dfrac{L - L_{G_3}}{B\omega_3 \cos\theta_2}.$

Q9 Pour que ~~θ_2~~ M_{01} et N_{01} soient faibles ...

| Pour N_{01} faible $\Rightarrow$ $\ddot{\theta}_2$ faible.
| Pour M_{01} faible $\Rightarrow$ θ_2 faible

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Q10 $t_{5\%} \simeq 0{,}01\ s < 0{,}033\ s$; $\varepsilon(\infty) = 0$

Très rapide et précis, on peut donc dire $H_2(p) = 1$
$= K_2$

Q11 $\theta_2 = \frac{1}{p} K_2 (U - C\theta_2) \Rightarrow p\theta_2 = K_2 U - K_2 C \theta_2$

$$H_{o2} = \frac{\theta_2}{U} = \frac{K_2}{p + K_2 C}$$

~~Avec $C(p) = K_{10}$~~ Entrée $U(p) = \frac{1}{p}$. $\theta_2(p) = \frac{K_2}{p(p+K_2C)}$

$$\lim_{t\to\infty} \theta_2(t) = \lim_{p\to 0} p\theta_2(p) = \lim_{p\to 0} \frac{K_2}{p+K_2C}$$

Avec $C(p) = K_{10}$, $\lim_{t\to\infty} \theta_2(t) = \frac{1}{K_{10}}$

Avec $C(p) = K_{10} + \frac{K_{11}}{p}$; $\lim_{t\to\infty} \theta_2(t) = 0$

Q12 $\Omega_2 = K_2 \left(U - \frac{C}{p}\Omega_2\right) \Rightarrow p\Omega_2 = K_2 U p - K_2 C \Omega_2$

$$H_m = \frac{\Omega_2}{U} = \frac{K_2 p}{p + K_2 C} = \frac{K_2 p}{p + K_2 K_{10} + \frac{K_2 K_{11}}{p}}$$

$$H_m = \frac{K_2 p^2}{p^2 + K_2 K_{10} p + K_2 K_{11}} = \frac{\frac{p^2}{K_{11}}}{\frac{p^2}{K_2 K_{11}} + \frac{K_{10}}{K_{11}} p + 1}$$

$$\omega_n = \sqrt{K_2 K_{11}} \Rightarrow K_{11} = \frac{\omega_n^2}{K_2} = 6\ rad/s.$$

$$\frac{2z}{\omega_n} = \frac{K_{10}}{K_{11}} \Rightarrow \frac{2z}{\sqrt{K_2 K_{11}}} = \frac{K_{10}}{K_{11}}$$

$$\Rightarrow K_{10} = \frac{2z\sqrt{K_{11}}}{\sqrt{K_2}} = 1{,}81\ rad/s.$$

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Q14 <u>On isole (1 + 2 + 3), TMD sur $(O_0, \vec{z_0})$</u>

Σ <u>Effort ext</u> : | Poids de m_1 et m_3
| Frottements visqueux en O_0

Q15 $\cancel{\vec{\delta}}$ $\vec{\sigma}(G_1, 1/0) = \begin{bmatrix} A_1 & & \\ & A_1 & \\ & & C_1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ 0 \\ 0 \end{bmatrix} = A_1 \dot{\theta}_1 \vec{x_1}$

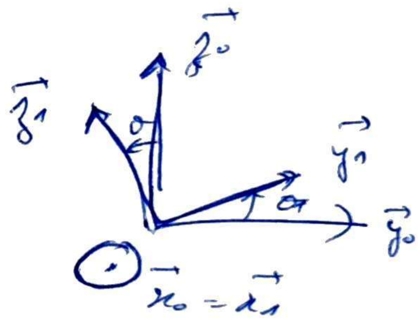
$$\vec{\delta}(G_1, 1/0) = A_1 \ddot{\theta}_1 . \vec{x_1}$$

$$\vec{\delta}(O_0, 1/0) = \vec{\delta}(G_1, 1/0) + \underbrace{\vec{O_0G_1}}_{L_{G1}\vec{z_1}} \wedge m_1 \vec{a}(G_1 \in 1/0)$$

$$\vec{O_0G_1} = L_{G1} \vec{z_1}$$

$$\vec{v}(G_1 \in 1/0) = -L_{G1} \dot{\theta}_1 \vec{y_1}$$

$$\vec{a}(G_1 \in 1/0) = L_{G1}\left(-\ddot{\theta}_1 \vec{y_1} - \dot{\theta}_1^2 \vec{z_1}\right)$$



$$\vec{\delta}(O_0, 1/0) = \left(A_1 \cancel{C_1} + m_1 L_{G1}^2\right) \ddot{\theta}_1$$

Q16 $\vec{O_0G_3} = L_{G3} . \vec{z_1} + H_{G3} \vec{y_1}$

$$\vec{v}(G_3 \in 3/0) = -L_{G3} \dot{\theta}_1 \vec{y_1} + H_{G3} \dot{\theta}_1 \vec{z_1}$$

$$\vec{a}(G_3 \in 3/0) = \left(-L_{G3} \ddot{\theta}_1 \cancel{\vec{z}} - H_{G3} \dot{\theta}_1^2\right) \vec{y_1} + \left(H_{G3} \ddot{\theta}_1 - L_{G3} \dot{\theta}_1^2\right) \vec{z_1}$$

Q17 $\vec{\delta}(G_3, 3/0) . \vec{x_0} = A_3 \ddot{\theta}_1 - B_3 \omega_3 \dot{\theta}_2$

$$\vec{\delta}(O_0, 3/0) . \vec{x_0} = \vec{\delta}(G_3, 3/0) . \vec{x_0} + \vec{O_0G_3} \wedge m_3 \vec{a}(G_3 \in 3/0)$$