1

# Correction DM de SI, MP, nov 23, Sismomètre

**Q1** $C_0 - k(\alpha - \alpha_0) - \cancel{\mu \dot{\alpha}} + M g_n d \sin\alpha = 0$

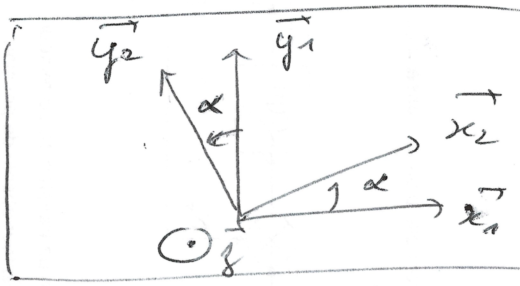
$\alpha = \alpha_0 = cte \longrightarrow C_0 + M \cdot g_n d \sin\alpha_0 = 0$

**Q2** $\Delta C = m g_n \dfrac{c}{2} \sin\alpha_0 = 10^{-3}$ Nm $>$ 0,5 Nmm

**Q3** $\vec{\sigma}(O_1 \in 2/0) = \bar{\bar{I}}_2(O_1) \cdot \vec{\Omega}_{2/0} + M \cdot \vec{O_1G} \wedge \vec{v}(O_1 \in 2/0)$

$$\vec{O_1G} = d \cdot \vec{y_2}$$

$$\vec{v}(O_1 \in 2/0) = V_x \vec{x_1} + V_y \cdot \vec{y_1}$$




$$\vec{\sigma}(O_1 \in 2/0) = I_{xx} \dot{\alpha} \vec{z} + M d (-V_x \cos\alpha - V_y \sin\alpha) \cdot \vec{z}$$

$$\vec{\sigma}(O_1 \in 2/0) = I_{xx} \dot{\alpha} \vec{z} - M d (V_x \cos\alpha + V_y \sin\alpha) \vec{z}$$

$$\vec{\delta}(O_1 \in 2/0) = \left( \frac{d \vec{\sigma}(O_1 \in 2/0)}{dt} \right)_0 + M \cdot \vec{v}(O_1 \in 2/0) \wedge \vec{v}(G \in 2/0)$$

**Q4**

$$\vec{v}(G \in 2/0) = \vec{v}(O_1 \in 2/0) + \vec{\Omega}_{2/0} \wedge \vec{O_1G} \qquad \vec{\Omega}_{2/0} = \dot{\alpha} \vec{z_1}$$

$$\vec{v}(G \in 2/0) = \vec{v}(O_1 \in 2/0) - d \dot{\alpha} \vec{x_2}$$

$$\vec{\delta}(O_1 \in 2/0) = I_{xx} \ddot{\alpha} \vec{z} - M d (\dot{V}_x \cos\alpha + \dot{V}_y \sin\alpha) \vec{z}$$

$$- M d (-V_x \dot{\alpha} \sin\alpha + V_y \dot{\alpha} \cos\alpha) \vec{z} - M d \dot{\alpha} (V_x \sin\alpha - V_y \cos\alpha) \vec{z}$$

$$\vec{\delta}(O_1 \in 2/0) = I_{xx} \ddot{\alpha} \vec{z} - M d \underbrace{(\dot{V}_x \cos\alpha + \dot{V}_y \sin\alpha)}_{\gamma_{x2}} \vec{z}$$

__Remarque :__ En utilisant HUYGENS

$$\bar{\bar{I}}_2(O_1) = \bar{\bar{I}}_2(G) + M \begin{bmatrix} d^2 & & \\ & 0 & \\ & & d^2 \end{bmatrix} \qquad \vec{OG} = d \vec{y_2}$$

2

$$\Rightarrow \overline{\overline{I}}_2(G) = \begin{bmatrix} I_{xx} - Md^2 & -I_{xy} & 0 \\ -I_{xy} & I_{yy} & 0 \\ 0 & 0 & J - Md^2 \end{bmatrix}$$

$$\vec{\sigma}(G \in 2/0) = \overline{\overline{I}}_2(G) . \vec{\Omega}^{2/0} = (I_{xx} - Md^2)\dot{\alpha}\vec{z}$$

$$\vec{\delta}(G \in 2/0) = (I_{xx} - Md^2)\ddot{\alpha}.\vec{z}$$

$$\vec{\delta}(O_1 \in 2/0) = \vec{\delta}(G \in 2/0) + \vec{O_1G} \wedge M.\vec{a}(G \in 2/0)$$

$$\vec{O_1G} = d.\vec{y_2}$$

$$\vec{v}(G \in 2/0) = V_x \vec{x_1} + V_y \vec{y_1} - d\dot{\alpha}\vec{x_2}$$

$$\vec{a}(G \in 2/0) = \dot{V}_x \vec{x_1} + \dot{V}_y \vec{y_1} - d\ddot{\alpha}\vec{x_2} - d\dot{\alpha}^2\vec{y_2}$$

$$\vec{\delta}(O_1 \in 2/0) = (I_{xx} - Md^2)\ddot{\alpha}\vec{z} - Md\dot{V}_x \cos\alpha\,\vec{z} - Md\dot{V}_y \sin\alpha\,\vec{z}$$
$$+ Md^2\ddot{\alpha}\vec{z}$$

<u>Finalement</u> : $\vec{\delta}(O_1 \in 2/0) = I_{xx}\ddot{\alpha}\vec{z} - Md(\dot{V}_x\cos\alpha + \dot{V}_y\sin\alpha)\vec{z}$

Q5 On isole (2), TMD en $O_1$

Q6 $\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\sin(\alpha_0 + \Delta\alpha) = \sin\alpha_0 \cos\Delta\alpha + \cos\alpha_0 \sin\Delta\alpha$

$\sin(\alpha_0 + \Delta\alpha) = \sin\alpha_0 + \Delta\alpha\cos\alpha_0$

de plus, d'après Q1 et $\alpha_{eq} = \alpha_0 \Rightarrow C_0 + Mg_n d\sin\alpha_0 = 0$

$(eq\,1) \Rightarrow J\Delta\ddot{\alpha} + \nu\Delta\dot{\alpha} + k\Delta\alpha = dMg_n(\sin\alpha_0 + \Delta\alpha\cos\alpha_0)$
$+ dM\gamma_{x_2} + C_0$

$\Rightarrow J\Delta\ddot{\alpha} + \nu\Delta\dot{\alpha} + k\Delta\alpha = dMg_n\cos\alpha_0\,\Delta\alpha + dM\gamma_{x_2}$

3

Q7 $$\frac{\alpha(p)}{\gamma_{x_2}(p)} = \frac{dm}{Jp^2 + \mu p + k - dm\, g_n \cos\alpha_0}$$

Stable si $k > dm\, g_n \cos\alpha_0$

Q8 $$\frac{\alpha(p)}{\gamma_{x_2}(p)} = \frac{\frac{dm}{k - dm\, g_n \cos\alpha_0}}{\frac{J}{k - dm\, g_n \cos\alpha_0} p^2 + \frac{\mu}{k - dm\, g_n \cos\alpha_0} p + 1}$$

$$\frac{\alpha(p)}{\gamma_{x_2}(p)} = \frac{A}{\frac{p^2}{\omega_0^2} + \frac{2\xi}{\omega_0} p + 1}$$

$$A = \frac{dm}{k - dm\, g_n \cos\alpha_0}$$

$$\omega_0 = \cancel{\sqrt{\frac{J}{k - dm\, g_n \cos\alpha_0}}} = \sqrt{\frac{k - dm\, g_n \cos\alpha_0}{J}}$$

$$\frac{2\xi}{\omega_0} = \frac{\mu}{k - dm\, g_n \cos\alpha_0}$$

$$\Longrightarrow \xi = \frac{\mu}{2\sqrt{J(k - dm\, g_n \cos\alpha_0)}}$$

Q9 $A$ max si $\cos\alpha_0$ max donc si $\alpha_0 = 0$