

(1)

Equilibrage

Q1 Torseur cinétique : $\{C^1_0\} = \left\{ \begin{array}{l} \vec{R}_c(1_0) \\ \vec{T}(O \in 1_0) \end{array} \right\}_O$

$\vec{R}_c(1_0)$: résultante cinétique

$\vec{T}(O \in 1_0)$: moment cinétique.

$$\left\{ \begin{array}{l} \vec{R}_c(1_0) = m \vec{v}(O \in 1_0) \\ \vec{OG} = x_G \vec{x}_1 + y_G \vec{y}_1 \Rightarrow \vec{v}(O \in 1_0) = x_G \dot{\theta} \vec{y}_1 \\ \vec{R}_c(1_0) = x_G \dot{\theta} m \vec{y}_1 \end{array} \right.$$

Calcul de $\vec{T}(O \in 1_0)$, on est en O ... cool ($\vec{v}(O) = \vec{0}$)

$$\vec{T}(O \in 1_0) = \begin{bmatrix} A & -F & -E \\ -F & B & -D \\ -E & -D & C \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \dot{\theta} \end{bmatrix} = \begin{array}{l} -E \dot{\theta} \vec{x}_1 \\ -D \dot{\theta} \vec{y}_1 \\ + C \dot{\theta} \vec{z}_1 \end{array}$$

Torseur dynamique : $\{D^1_0\} = \left\{ \begin{array}{l} \vec{R}_d(1_0) \\ \vec{\delta}(O \in 1_0) \end{array} \right\}_O$

$\vec{R}_d(1_0)$: résultante dynamique

$\vec{\delta}(O \in 1_0)$: moment dynamique

$$\vec{R}_d(1_0) = m \vec{a}(O \in 1_0)$$

$$\begin{aligned} \vec{R}_d(1_0) &= m (x_G \ddot{\theta} \vec{y}_1 - x_G \dot{\theta}^2 \vec{x}_1) \\ &= m x_G (\ddot{\theta} \vec{y}_1 - \dot{\theta}^2 \vec{x}_1) \end{aligned}$$

$$\begin{aligned} \vec{\delta}(0 \in \gamma_0) &= -E \ddot{\theta} \vec{x}_1 - E \dot{\theta}^2 \vec{y}_1 - D \ddot{\theta} \vec{y}_1 + D \dot{\theta}^2 \vec{x}_1 + C \ddot{\theta} \vec{y}_1 \\ \vec{\delta}(0 \in \gamma_0) &= (-E \ddot{\theta} + D \dot{\theta}^2) \vec{x}_1 - (E \dot{\theta}^2 + D \ddot{\theta}) \vec{y}_1 + C \ddot{\theta} \vec{y}_1 \end{aligned}$$

(Q2) BALPE : Bilan des Actions ... (sous forme de torseur)

$$\left\{ \mathcal{T}_{\text{pivot}} \right\}_{0 \rightarrow 1} = \left\{ \begin{array}{cc} X & L \\ Y & \Pi \\ Z & 0 \end{array} \right\}_0 \quad \left\{ \mathcal{T}_{\text{moteur}} \right\}_{\rightarrow 1} = \left\{ \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ 0 & \text{cin} \end{array} \right\}_0$$

$$\left\{ \mathcal{T}_{\text{pes}} \rightarrow 1 \right\} = \left\{ \begin{array}{cc} 0 & 0 \\ 0 & 0 \\ -mg & 0 \end{array} \right\}_G$$

$$\vec{\Pi}(0) = \vec{\Pi}(G) + \vec{OG}_1 \wedge \vec{P} = \vec{0} + (x_G \vec{x}_1 + z_G \vec{z}_0) \wedge (-mg \vec{z}_0)$$

$$\vec{\Pi}(0) = mg x_G \vec{y}_1 \quad \leftarrow \text{(on retrouve bra} \times \text{bras de levier)}$$

$$\left\{ \mathcal{T}_{\text{pes}} \rightarrow 1 \right\} = \left\{ \begin{array}{cc} 0 & 0 \\ 0 & mg x_G \\ -mg & 0 \end{array} \right\}_0$$

(Q3) PFD \Rightarrow

$$\left| \begin{array}{l} -m_1 x_G \dot{\theta}^2 = X \\ m_1 x_G \ddot{\theta} = Y \\ 0 = Z - mg \\ -E \ddot{\theta} + D \dot{\theta}^2 = L \\ -E \dot{\theta}^2 - D \ddot{\theta} = \Pi + mg x_G \\ C \ddot{\theta} = \text{cin} \end{array} \right.$$

③ Q4 On en déduit $\left\{ \begin{array}{l} X = \dots ; Y = \dots, Z = \dots \\ L = \dots ; \Pi = \dots \end{array} \right.$

Q5 On veut $X = Y = L = \Pi = 0$, il faut :

- ① $x_0 = 0$
- ② $E = D = 0$

Q6 On ajoute 2 masses ponctuelles : m_1 et m_2

$$\left. \begin{array}{l} \vec{Ob}_1 = x_1 \vec{x}_1 + y_1 \vec{y}_1 + z_1 \vec{z}_0 \\ \vec{Ob}_2 = x_2 \vec{x}_1 + y_2 \vec{y}_1 + z_2 \vec{z}_0 \end{array} \right\} \begin{array}{l} 8 \text{ paramètres} \\ (m_1, m_2, x_1, \dots) \end{array}$$

Centre de masse de l'ensemble :

$$\vec{Ob}_E = \frac{m \vec{Ob} + m_1 \vec{Ob}_1 + m_2 \vec{Ob}_2}{m + m_1 + m_2} = \vec{0}$$

$$\Rightarrow \left\{ \begin{array}{l} m x_0 + m_1 x_1 + m_2 x_2 = 0 \\ m_1 y_1 + m_2 y_2 = 0 \end{array} \right. \quad 2 \text{ équations}$$

Matrice d'inertie de la masse m_1 en O (Huygens)

$$\overline{\overline{I}}(m_1, 0) = \begin{bmatrix} A_1 & -F_1 & -E_1 \\ & B_1 & -D_1 \\ & & C_1 \end{bmatrix} \quad \begin{array}{l} D_1 = m_1 y_1^2 z_1^2 \\ E_1 = m_1 x_1 z_1^2 \\ F_1 = m_1 x_1 y_1 z_1 \end{array}$$

$$\text{Donc : } \left. \begin{array}{l} D_E = D + m_1 y_1 z_1^2 + m_2 y_2 z_2^2 = 0 \\ E_E = E + m_1 x_1 z_1^2 + m_2 x_2 z_2^2 = 0 \end{array} \right\} 2 \text{ équations}$$

④ Bilan : on a | 8 inconnues ($m_1, m_2, x_1, x_2, y_1, \dots$)
| 6 équations.

On rajoute des équations :

$$\left\{ \begin{array}{l} z_1 = 0 \\ z_2 = L \\ \sqrt{(x_1)^2 + (y_1)^2} = R \\ \sqrt{(x_2)^2 + (y_2)^2} = R \end{array} \right.$$

↳ Paves placés à une distance R de
l'axe de rotation.
Et ...