Corrigé STATIQUE : Modélisation des actions mécaniques

Exercice 1 Torseur résultant

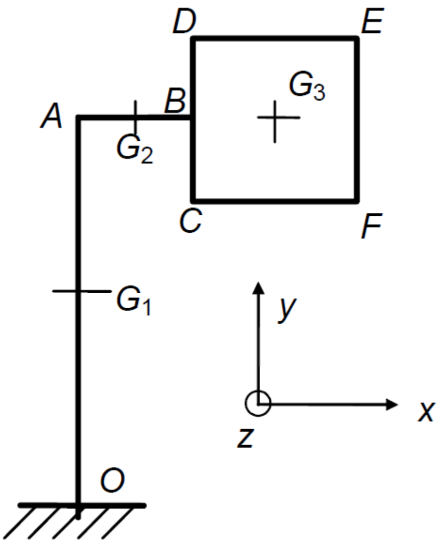
$$\{\tau FA\} = \left\{ \begin{matrix} \vec{F}_A \\ \vec{M}_{FA}(O) \end{matrix} \right\}_O = \left\{ \begin{matrix} 100.\vec{x} \\ -200.\vec{z} \end{matrix} \right\}_O$$

$$\{\tau FB\} = \left\{ \begin{matrix} \vec{F}_B \\ \vec{M}_{FB}(O) \end{matrix} \right\}_O = \left\{ \begin{matrix} -50.\vec{x} + 100.\vec{y} \\ 250.\vec{z} \end{matrix} \right\}_O$$

$$\{\tau FA + FB\} = \left\{ \begin{matrix} \vec{F} \\ \vec{M}\ (O) \end{matrix} \right\}_O = \left\{ \begin{matrix} 50.\vec{x} + 100.\vec{y} \\ 50.\vec{z} \end{matrix} \right\}_O$$

Support : Droite d'équation $y = 2.x - 1$

Exercice 2 Panneau indicateur



$$\{\tau P1\} = \left\{ \begin{matrix} -3000.\vec{y} \\ \vec{0} \end{matrix} \right\}_O$$

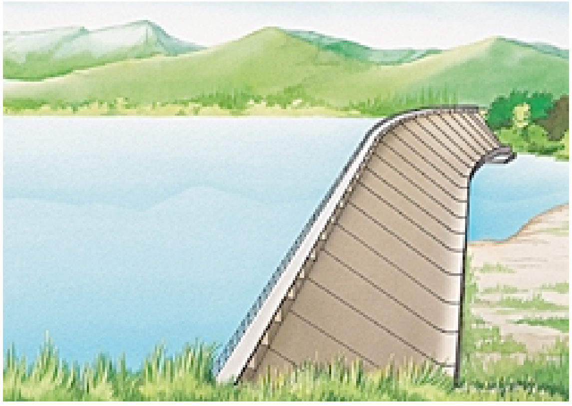
$$\{\tau P2\} = \left\{ \begin{matrix} -1000.\vec{y} \\ -1500.\vec{z} \end{matrix} \right\}_O$$

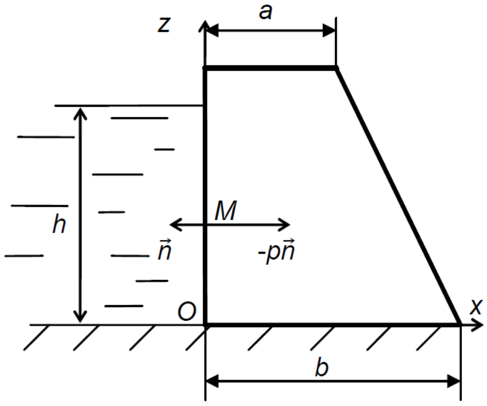
$$\{\tau P3\} = \left\{ \begin{matrix} -4000.\vec{y} \\ -20000.\vec{z} \end{matrix} \right\}_O \qquad \{\tau Vent\} = \left\{ \begin{matrix} -6000.\vec{z} \\ -45000.\vec{x} + 30000.\vec{y} \end{matrix} \right\}_O$$

$$\{\tau\} = \left\{ \begin{matrix} -8000.\vec{y} - 6000.\vec{z} \\ -45000.\vec{x} + 30000.\vec{y} - 21500.\vec{z} \end{matrix} \right\}_O$$

Exercice 3

Barrage





Force élémentaire : $$d\vec{F} = p.dS.\vec{x} = \rho.g.(h-z).l.dz.\vec{x}$$

On intègre et on trouve : $$\vec{F} = \rho.g.l.\frac{h^2}{2}.\vec{x}$$

Remarque : F = Surface*pression moyenne

Moment élémentaire en O : $$d\vec{M}(O) = d\vec{M}(M) + \overrightarrow{OM} \wedge d\vec{F}$$

$$d\vec{M}(O) = z.\vec{z} \wedge \rho.g.(h-z).l.dz.\vec{x} = \rho.g.(h.z-z^2).l.dz.\vec{y}$$

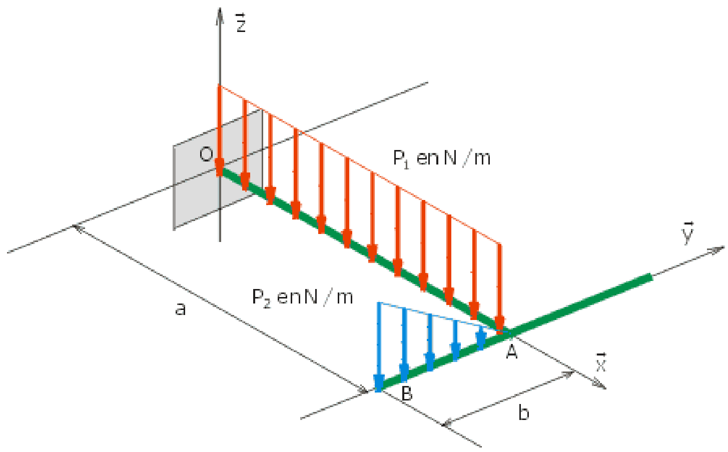
On intègre et on trouve : $$\vec{M}(O) = \rho.g.l.\frac{h^3}{6}.\vec{y}$$

Centre de poussée : on cherche un point A tel que $\vec{M}(A) = \vec{0}$

$$\vec{M}(A) = \vec{M}(O) + \overrightarrow{AO} \wedge \vec{F} = \vec{0} \quad \Rightarrow \quad \dots \quad \Rightarrow \quad \overrightarrow{OA} = \frac{h}{3}.\vec{z}$$

Résultat classique d'une « répartition triangulaire »

Exercice 4 Poutre



$$\left\{\tau P_1 \rightarrow Poutre\right\} = \left\{\begin{matrix}\vec{F}_1 \\ \vec{M}_1(0)\end{matrix}\right\}_O = \left\{\begin{matrix}-P_1.a.\vec{z} \\ +\dfrac{P_1.a}{2}.\vec{y}\end{matrix}\right\}_O$$

$$\left\{\tau P_2 \rightarrow Poutre\right\} = \left\{\begin{matrix}\vec{F}_2 \\ \vec{M}_2(M)\end{matrix}\right\}_M = \left\{\begin{matrix}-\dfrac{P_2.b}{2}.\vec{z} \\ \vec{0}\end{matrix}\right\}_M$$

M situé à 1/3 2/3 de AB

$$\vec{M}_2(O) = \vec{M}_2(M) + \overrightarrow{OM} \wedge \vec{F}_2 = 0 + (a.\vec{x} - \frac{2}{3}.b.\vec{y}) \wedge -\frac{P_2.b}{2}.\vec{z}$$

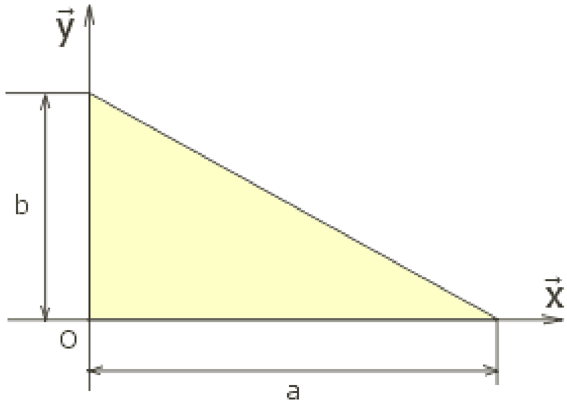
$$\vec{M}_2(O) = \frac{P_2}{2}.(a.b.\vec{y} + \frac{2}{3}.b^2.\vec{x})$$

$$\left\{\tau P_2 \rightarrow Poutre\right\} = \left\{\begin{matrix}\vec{F}_2 \\ \vec{M}_2(O)\end{matrix}\right\}_O = \left\{\begin{matrix}-\dfrac{P_2.b}{2}.\vec{z} \\ \dfrac{P_2}{2}.(a.b.\vec{y} + \dfrac{2}{3}.b^2.\vec{x})\end{matrix}\right\}_O$$

$$\left\{\tau P \rightarrow Poutre\right\} = \left\{\begin{matrix}\vec{F} \\ \vec{M}(0)\end{matrix}\right\}_O = \left\{\begin{matrix}\vec{F}_1 + \vec{F}_2 \\ \vec{M}_1(O) + \vec{M}_2(O)\end{matrix}\right\}_O$$

Exercice 5

Triangle rectangle.



Intuitivement on peut dire :

$$\overrightarrow{OG} = \frac{1}{3}.(a.\vec{x} + b.\vec{y})$$

Par calcul : $\overrightarrow{OG} = \frac{1}{V}\int_V \overrightarrow{OM}.dv$

$$x_G = \frac{1}{S}\int_S x.ds = \frac{2}{a.b}\int_0^a x.(-\frac{b}{a}.x + b).dx = ... = \frac{a}{3}$$

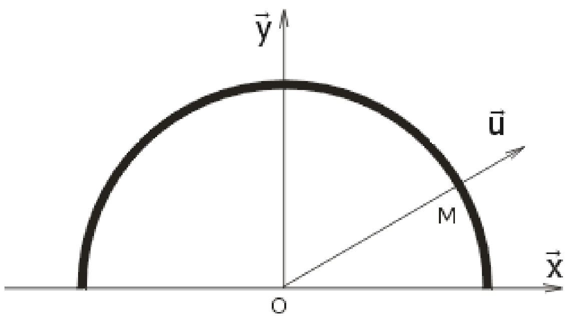
$$\{\tau poids \to S\} = \begin{Bmatrix} \vec{F} \\ \vec{M}(G) \end{Bmatrix}_G = \begin{Bmatrix} -m.g.\vec{z} \\ \vec{0} \end{Bmatrix}_G = \begin{Bmatrix} \vec{F} \\ \vec{M}(O) \end{Bmatrix}_O$$

Directement : $\vec{M}(O) = -\frac{m.g.a}{3}.\vec{z}$

Ou par calcul : $\vec{M}(O) = \vec{M}(G) + \overrightarrow{OG} \wedge \vec{F}$

Exercice 6

Demi-cercle.



$$\overrightarrow{OG} = \frac{1}{V}\int_V \overrightarrow{OM}.dv$$

$$y_G = \frac{1}{L}\int_L \overrightarrow{OM}.\vec{y}.dl = \frac{1}{\pi.r}.\int_L r.\vec{u}.\vec{y}.r.d\theta$$

$$y_G = \frac{1}{\pi.r}.\int_0^\pi r.\vec{u}.\vec{y}.r.d\theta = \frac{r}{\pi}.\int_0^\pi \sin\theta.d\theta$$

$$y_G = \frac{2.r}{\pi}$$