

Comparaisons

P S I · 2 4 2 5 · B O N U S

Comparaisons

Considérons $\alpha, \beta, \gamma, \delta$ dans $\mathbb{R}_+^*$.

Pour n entier au voisinage de $+\infty$,

$$\ln^\alpha(n) \ll n^\beta \ll e^{\gamma n} \ll n! \ll n^n.$$

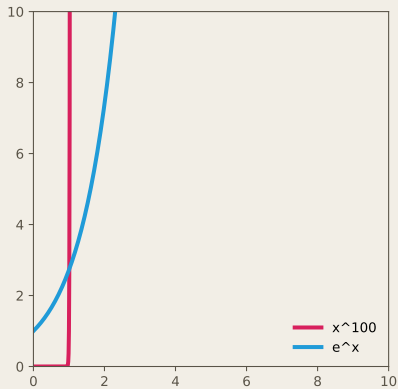
Pour x réel au voisinage de $+\infty$,

$$\ln^\alpha(x) \ll x^\beta \ll e^{\gamma x} \ll \Gamma(x) \ll x^x.$$

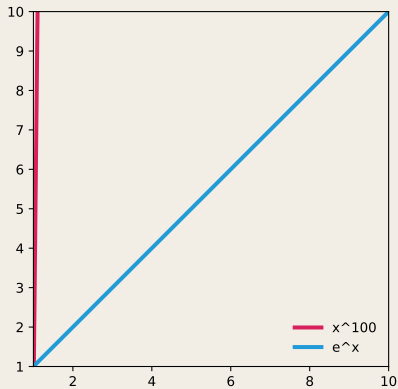
En particulier,

$$x^{100} \ll_{+\infty} e^x$$

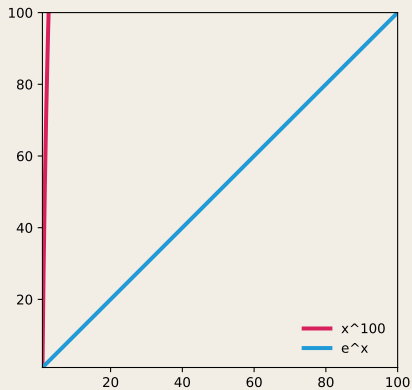
Mais...



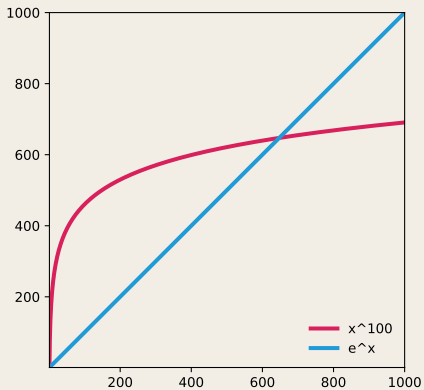
Mais...



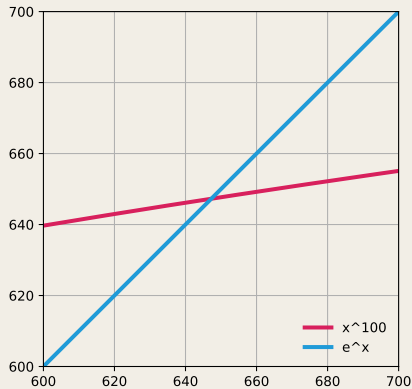
Mais...



Mais...



Mais...



Dichotomie entre 600 et 700

```
from math import log

a, b = 600, 700
eps = 0.001

def f(x):
    return 100 * log(x) - x

while b - a > eps:
    m = (a + b) / 2
    if f(m) * f(b) > 0:
        b = m
    else:
        a = m
print('La solution est comprise\nentre', \
      a, '\net', b)
```

Dichotomie entre 600 et 700

La solution est comprise
entre 647.277069092
et 647.277832031

Ainsi,

en posant $a = 647,277$, on a

$$a^{100} \approx e^a \approx 1,28 \cdot 10^{281}.$$