

THE CRONE SUSPENSION

A. Oustaloup, X. Moreau and M. Nouillant

*Equipe CRONE-LAP-ENSERB, Université Bordeaux I, 351, cours de la Libération, 33405 Talence Cédex, France
(oustalou@lap.u-bordeaux.fr)*

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Abstract: This paper deals with robust control and advanced suspension of vehicles, and more specifically, with a new system called the CRONE suspension, based on non-integer derivation. The CRONE suspension results from a traditional suspension model whose spring and damper are replaced by a mechanical and hydropneumatic system defined by a non-integer order force-displacement transmittance. This system is called the CRONE suspension because of the link with the second-generation CRONE control, i.e. the vertical template. That is why the principle of the second-generation CRONE control is used to synthesise the CRONE suspension's transmittance. The suspension parameters are determined from a constrained optimisation of a performance criterion. A two-degree-of-freedom quarter-car model is used to evaluate performance. The frequency and time responses, for various values of the vehicle load, reveal a great robustness of the degree of stability through the constancy of the resonance ratio in the frequency domain, and of the damping ratio in the time domain.

Keywords: robust control, second-generation CRONE control, advanced suspensions, CRONE suspension, robustness of the degree of stability.

1. INTRODUCTION

Automotive suspensions are designed to provide good vibration insulation for the passengers, and to maintain adequate adherence of the wheels for braking, acceleration and handling.

The introduction of electronics in automotive suspensions has been considered for decades, but it is only recently that the automotive industry has begun to seriously consider modulated, semi-active and active suspensions. Many reports (Rakheja and Ahmed, 1991; Yasuda and Doi, 1991; Jezequel and Roberti, 1992) have shown that the introduction of active or semi-active elements in the suspension increases vehicle performance, even given the ride comfort/road-holding ability dilemma (Hedrick and Butsuen, 1990), using an optimal approach (Daver, *et al.*, 1992; Moreau, *et al.*, 1993). The improvement is not only due to progress in hydraulics and electronics (Oustaloup and Nouillant, 1990; Dunwoody, 1991; Gohring and Von Glasner, 1993) but also thanks to control-law research on the active or semi-active elements of suspension systems (Redfield and Karnopp, 1989; Yue, *et al.*, 1989; Moreau, *et al.*, 1996).

This paper deals with a robust control law for a suspension system which develops a force that is proportional to the non-integer derivative of its relative displacement. This system is called the CRONE suspension because of the link with the second-generation CRONE control (Commande Robuste d'Ordre Non Entier) (Oustaloup, 1991), i.e. the template which characterizes both the control and the suspension.

The paper is divided into five parts. Section 1 gives the principles of the CRONE suspension, and of its model. Section 2 develops the synthesis method of the suspension. Section 3 describes the constrained optimisation used to determine the parameters of the CRONE suspension. Section 4 examines the frequency and time responses which show the robustness of both the resonance and damping ratios versus load variation. Finally, in Section 5 conclusions are given.

2. MODELING

The quarter-car model shown in Fig. 1 has been studied by many authors, to analyse and optimise automotive suspensions.

The CRONE suspension results from a traditional suspension model whose spring and damper are replaced by a mechanical and hydropneumatic system (Fig. 1), defined by a non-integer order transmittance. m_2 is the mass supported by each wheel, and is taken as equal to a quarter of the total mass of the body. k_2 is the stiffness of the spring, and b_2 the damping coefficient for a traditional suspension. $C(s)$ is the CRONE suspension transmittance replacing the traditional suspension transmittance. k_1 is the stiffness and b_1 the damping coefficient of the tyre. m_1 is the unsprung mass. $z_0(t)$ is the deflexion of the road, and $z_1(t)$ and $z_2(t)$ are the vertical displacements of the wheel and body respectively.

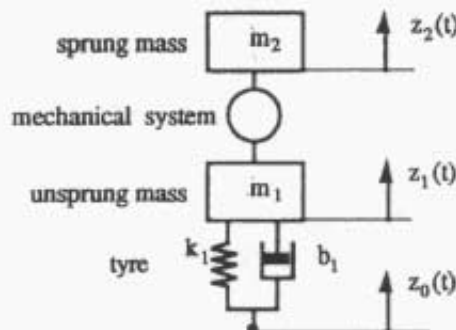


Fig. 1. Two degree-of-freedom model

The suspension develops a force $F_2(s)$ which is a function of the relative displacement $Z_{12}(s)$, and which obeys symbolically the general relation:

$$F_2(s) = C(s) Z_{12}(s), \quad (1)$$

with

$$Z_{12}(s) = Z_1(s) - Z_2(s). \quad (2)$$

If it is assumed that the tyre does not leave the ground and that $z_1(t)$ and $z_2(t)$ are measured from the static equilibrium position, then the application of the fundamental law of dynamics leads to the linearised equations of motion:

$$m_1 \ddot{z}_1(t) = f_1(t) - f_2(t) \quad (3)$$

and

$$m_2 \ddot{z}_2(t) = f_2(t), \quad (4)$$

in which

$$f_1(t) = k_1 (z_0(t) - z_1(t)) + b_1 (\dot{z}_0(t) - \dot{z}_1(t)) \quad (5)$$

and $f_2(t)$ is the force developed by the suspension.

The Laplace transform of equations (3), (4) and (5), assuming zero initial conditions, are

$$m_1 s^2 Z_1(s) = k_1 (Z_0(s) - Z_1(s)) + b_1 s (Z_0(s) - Z_1(s)) - F_2(s) \quad (6)$$

and

$$m_2 s^2 Z_2(s) = F_2(s), \quad (7)$$

or, given equation (1) of $F_2(s)$,

$$m_1 s^2 Z_1(s) = k_1 (Z_0(s) - Z_1(s)) + b_1 s (Z_0(s) - Z_1(s)) - C(s) (Z_1(s) - Z_2(s)) \quad (8)$$

and

$$m_2 s^2 Z_2(s) = C(s) (Z_1(s) - Z_2(s)). \quad (9)$$

To analyse the vibration insulation of the sprung mass, two transmittances are defined:

$$T_2(s) = \frac{Z_2(s)}{Z_1(s)} = \frac{C(s)}{m_2 s^2 + C(s)} \quad (10)$$

and

$$S_2(s) = \frac{Z_{12}(s)}{Z_1(s)} = \frac{m_2 s^2}{m_2 s^2 + C(s)}. \quad (11)$$

To study ride comfort and road holding ability, three additional transmittances are defined:

$$H_s(s) = \frac{A_2(s)}{V_0(s)}, H_{12}(s) = \frac{Z_{12}(s)}{V_0(s)}, H_{01}(s) = \frac{Z_{01}(s)}{V_0(s)}, \quad (12)$$

in which $A_2(s)$ is acceleration of the sprung mass, $Z_{12}(s)$ suspension deflection, $Z_{01}(s)$ tyre deflection and $V_0(s)$ road input vertical velocity. A commonly used road input model is that $v_0(t)$ is white noise whose intensity is proportional to the product of the vehicle's forward speed and a road roughness parameter.

3. SYNTHESIS METHOD OF THE CRONE SUSPENSION

The synthesis method of the CRONE suspension is based on the interpretation of transmittances $T_2(s)$ and $S_2(s)$ which can be written as:

$$T_2(s) = \frac{\beta(s)}{1 + \beta(s)} \quad (13)$$

and

$$S_2(s) = \frac{1}{1 + \beta(s)}, \quad (14)$$

in which

$$\beta(s) = \frac{C(s)}{m_2 s^2}. \quad (15)$$

The transmittances $T_2(s)$ and $S_2(s)$ can here be considered to be those of an elementary control loop whose $\beta(s)$ is the open-loop transmittance.

Given that relation (15) expresses that a variation of mass is accompanied by a variation of open-loop gain, the principle of the second-generation CRONE control (Oustaloup, 1991) can be used by synthesising the open-loop Nichols locus which traces a vertical template for the nominal mass.

3.1 First version of the CRONE suspension

A first way of synthesising the Nichols locus, defined in Fig. 2, consists of determining an open-loop transmittance which presents an asymptotic behaviour of order n between 1 and 2.

This asymptotic behaviour can be obtained with a transmittance of the form:

$$\beta(s) = \left(\frac{\omega_u}{s} \right)^n. \quad (16)$$

Identification of equations (15) and (16) leads to:

$$C(s) = \left(\frac{s}{\omega_0} \right)^m \quad (17)$$

in which

$$m = 2 - n \in]0,1[\quad (18)$$

and

$$\omega_0 = \frac{1}{(m_2 \omega_u^{2-m})^{1/m}} \quad (19)$$

Equation (1) then becomes:

$$F_Z(s) = \left(\frac{s}{\omega_0} \right)^m [Z_1(s) - Z_2(s)] \quad (20)$$

namely, in the time domain:

$$f_Z(t) = \frac{1}{\omega_0^m} \left(\frac{d}{dt} \right)^m [z_1(t) - z_2(t)] \quad (21)$$

Equation (21), thus obtained, expresses that the CRONE suspension develops a force which is proportional to the non-integer derivative of its relative displacement. The non-integer order is between 0 and 1 (Oustaloup, *et al.*, 1993).

3.2 Second version of the CRONE suspension

Another way of synthesising the open-loop Nichols locus, given that robustness does not require an infinitely long template, consists of determining a transfer $\beta(s)$ which successively presents (Fig.3):

- an order-2 asymptotic behaviour at low frequencies to eliminate tracking error;
- an order- n asymptotic behaviour, where n is between 1 and 2, exclusively around frequency ω_u , to limit the synthesis of the non-integer derivation over a truncated frequency interval;
- an order-1 asymptotic behaviour at high frequencies, to ensure satisfactory filtering of vibrations at high frequencies.

Such localised behaviour can be obtained with a transmittance of the form:

$$\beta(s) = C_0 \frac{\left(1 + \frac{s}{\omega_b}\right)^m}{\left(1 + \frac{s}{\omega_h}\right)^{m-1}} \left(\frac{\omega_0}{s}\right)^2 \quad (22)$$

in which:

$$\omega_b \ll \omega_A, \omega_B \ll \omega_h \text{ and } m = 2 - n \in]0,1[\quad (23)$$

Identification of equations (15) and (22) gives:

$$1/\sqrt{m_2} = \omega_0 \quad (24)$$

and

$$C(s) = C_0 \frac{\left(1 + \frac{s}{\omega_b}\right)^m}{\left(1 + \frac{s}{\omega_h}\right)^{m-1}} \quad (25)$$

The equation thus obtained defines the ideal version of the suspension. The corresponding real version (Oustaloup, 1991) is defined by a transfer of integer order:

$$C_N(s) = C_0 \frac{\prod_{i=1}^N \left(1 + \frac{s}{\omega_i}\right)}{\prod_{i=1}^{N-1} \left(1 + \frac{s}{\omega_i}\right)} \quad (26)$$

in which:

$$\frac{\omega_{i+1}}{\omega_i} = \frac{\omega_{i+1}}{\omega_i} = \alpha\eta > 1; \quad \frac{\omega_i}{\omega_i} = \alpha; \quad \frac{\omega_{i+1}}{\omega_i} = \eta; \quad \alpha\eta = \left(\frac{\omega_h}{\omega_b}\right)^{1/N}; \quad (27)$$

$$\alpha = (\alpha\eta)^m; \quad \omega_1 = \omega_b \eta^{1/2} \text{ and } \omega_N = \omega_h \eta^{-1/2},$$

with N number of cells.

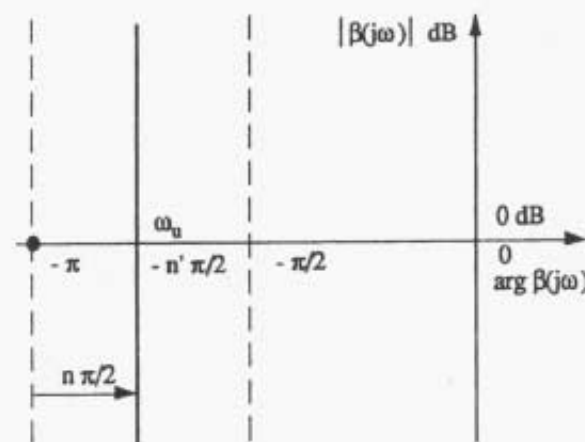


Fig. 2. Open-loop Nichols locus of the first-version CRONE suspension

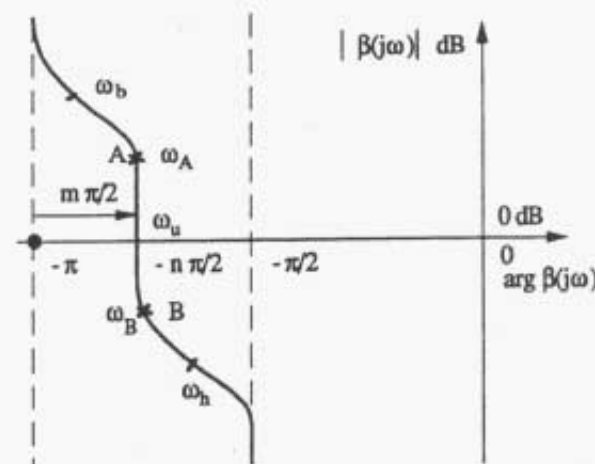


Fig. 3. Open-loop Nichols locus of the second-version CRONE suspension

4. DETERMINATION OF CRONE SUSPENSION PARAMETERS: CONSTRAINED OPTIMISATION

By defining the transmittances (12) with respect to $v_0(t)$, all frequencies contribute equally to their mean-square values. That is why the determination of CRONE suspension parameters is based on the minimisation of a criterion J , composed of the H_2 -norm

of the transmittances $H_a(j\omega)$, $H_{12}(j\omega)$ and $H_{01}(j\omega)$, namely:

$$J = \frac{\rho_1}{\lambda_1} \int_{\omega_0}^{\omega_h} |H_a(j\omega)|^2 d\omega + \frac{\rho_2}{\lambda_2} \int_{\omega_0}^{\omega_h} |H_{12}(j\omega)|^2 d\omega + \frac{\rho_3}{\lambda_3} \int_{\omega_0}^{\omega_h} |H_{01}(j\omega)|^2 d\omega + \frac{\rho_4}{\lambda_4} \int_{\omega_0}^{\omega_h} |H(j\omega)|^2 d\omega, \quad (28)$$

in which ρ_i are the weighting factors, λ_i the H_2 -norm computed for the traditional suspension and $H(j\omega)$ the transmittance between force $F_2(j\omega)$ developed by the suspension and the road input velocity $V_0(j\omega)$, namely:

$$H(j\omega) = \frac{F_2(j\omega)}{V_0(j\omega)} = m_2 H_a(j\omega). \quad (29)$$

To obtain a significant comparison between traditional and CRONE suspension performances, two constraints are fixed for the minimal sprung mass:

- equal unit gain frequency of open loop $\beta(j\omega)$;
- equal resonance ratio of transmittance $T_2(j\omega)$.

For the traditional suspension, the expression of unit gain frequency ω_{u2} and resonance ratio Q_2 are given by

$$\omega_{u2} = \frac{\sqrt{b_2^2 + \sqrt{b_2^4 + 4 m_2^2 k_2^2}}}{\sqrt{2} m_2} \quad (30)$$

and

$$Q_2 = \frac{2 \sqrt{2} \zeta_2^2}{\sqrt{\sqrt{1 + 8 \zeta_2^2} - 1 - 4 \zeta_2^2 + 8 \zeta_2^4}}, \quad (31)$$

$$\text{in which } \zeta_2 = \frac{b_2}{2 \sqrt{k_2 m_2}}. \quad (32)$$

For the CRONE suspension, the expression of unit gain frequency ω_u and resonance ratio Q are given by (Moreau, 1995)

$$\omega_u = \frac{1}{(m_2 \omega_0^2)^{1/(2-m)}} \quad (33)$$

and

$$Q = \frac{1}{\sin \left[(2-m) \frac{\pi}{2} \right]}. \quad (34)$$

5. PERFORMANCE

5.1 Example

The passive suspension used for comparison is the rear hydropneumatic suspension of a Citroën BX. The parameters of the quarter-car model are given by :

- sprung mass: $150 \text{ kg} \leq m_2 \leq 300 \text{ kg}$;
- unsprung mass: $m_1 = 24 \text{ kg}$;
- stiffness of tyre: $k_1 = 273 \, 820 \text{ N/m}$;
- damping of tyre: $b_1 = 50 \text{ Ns/m}$;

- for the passive hydropneumatic suspension
- stiffness: $k_2 = 2 \, 500 \text{ N/m}$;
- damping coefficient: $b_2 = 850 \text{ Ns/m}$.

From this data, the constrained optimisation of the criterion J , computed using the optimisation toolbox of Matlab (Matlab, 1992), provides the optimal parameters of the first-version CRONE suspension, namely:

$$m = 0.8 \text{ et } \omega_0 = 12.16 \cdot 10^{-5} \text{ rd/s}. \quad (35)$$

The optimal parameters of the second-version CRONE suspension are:

- for the ideal version:

$$m = 0.8; \quad C_0 = 215; \quad (36)$$

$$\omega_b = 0.1 \text{ rd/s}; \quad \omega_h = 200 \text{ rd/s};$$

- for the real version:

$$N = 5; \quad C_0 = 215;$$

$$\alpha = \omega_1/\omega_1' = 3.37; \quad \eta = \omega_{i+1}/\omega_i = 1.355;$$

$$\omega_1' = 0.1164 \text{ rd/s}; \quad \omega_1 = 0.4714 \text{ rd/s};$$

$$\omega_2' = 0.5324 \text{ rd/s}; \quad \omega_2 = 1.796 \text{ rd/s};$$

$$\omega_3' = 2.435 \text{ rd/s}; \quad \omega_3 = 8.215 \text{ rd/s};$$

$$\omega_4' = 11.137 \text{ rd/s}; \quad \omega_4 = 37.57 \text{ rd/s};$$

$$\omega_5' = 50.92 \text{ rd/s}. \quad (37)$$

5.2 Frequency responses

Figures 4, 5 and 6 show the frequency performances in open loop and closed loop.

Figure 4 gives the Nichols loci $\beta(j\omega)$ for the traditional and CRONE suspensions. The phase margin varies with mass m_2 for the traditional suspension. On the other hand, the phase margin is independent for the CRONE suspension, where the Nichols loci in the open loop trace the template which characterizes the second-generation CRONE control.

Figures 5 and 6 give the gain diagrams of $T_2(j\omega)$ and $S_2(j\omega)$ for traditional and CRONE suspensions. For the CRONE suspension, the resonance ratio can be seen to be both weak and insensitive to variations of mass m_2 . This shows a better robustness of the CRONE suspension in the frequency domain.

5.3 Step responses

Figures 7 and 8 show the step responses of the car body and the wheel for both suspensions. For the CRONE suspension it can be seen that the first overshoot remains constant, showing a better robustness for the CRONE suspension in the time domain (Fig. 7.b). The road-holding ability is the same for both suspensions (Fig. 8).

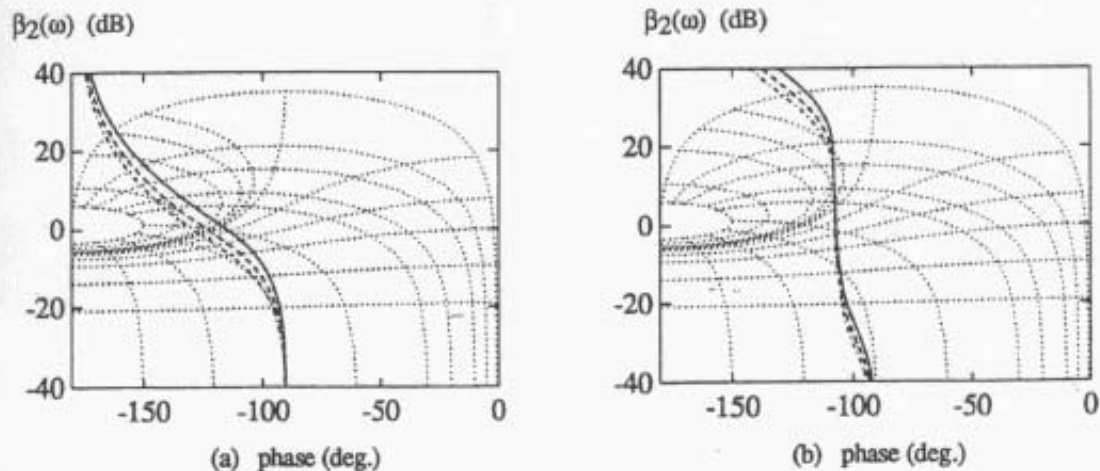


Fig. 4. Nichols loci in open loop for (a) traditional and (b) CRONE suspensions:
(——) $m_2 = 150$ kg; (----) $m_2 = 225$ kg; (- · - · -) $m_2 = 300$ kg

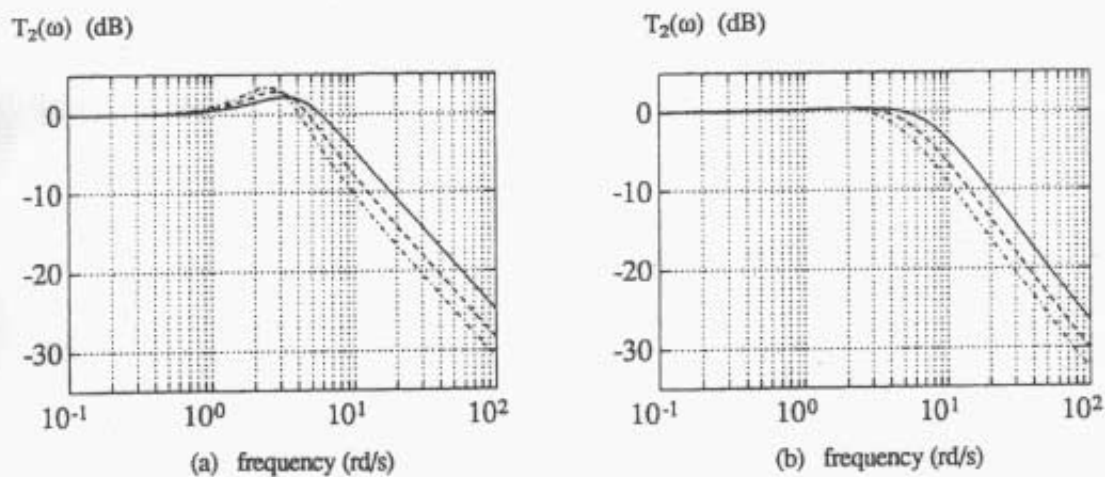


Fig. 5. Gain diagrams of $T_2(j\omega)$ for traditional (a) and CRONE (b) suspensions:
(——) $m_2 = 150$ kg; (----) $m_2 = 225$ kg; (- · - · -) $m_2 = 300$ kg

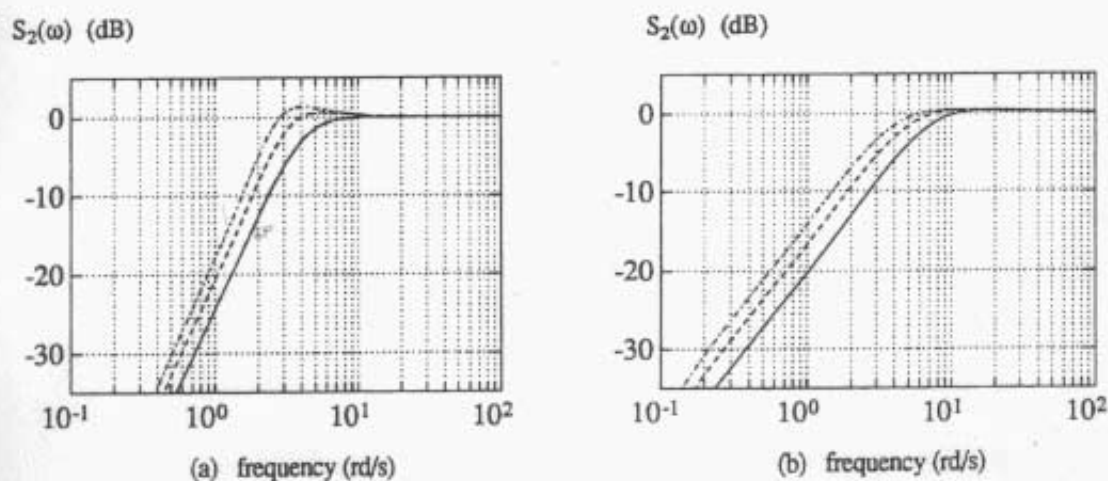
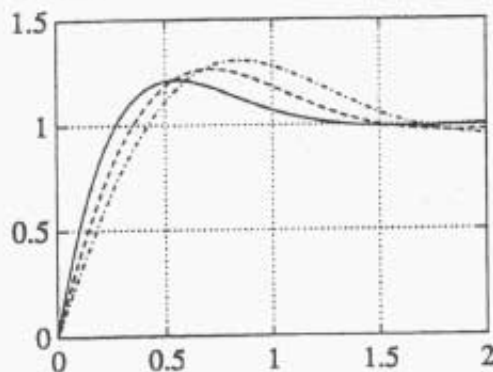
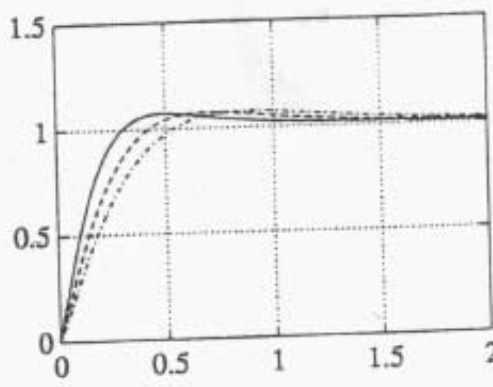


Fig. 6. Gain diagrams of $S_2(j\omega)$ for traditional (a) and CRONE (b) suspensions:
(——) $m_2 = 150$ kg; (----) $m_2 = 225$ kg; (- · - · -) $m_2 = 300$ kg

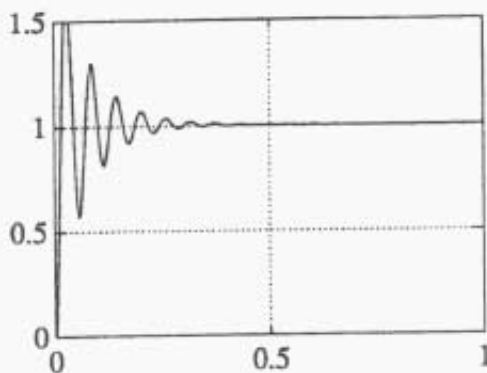
$z_2(t)$ (cm)

(a) time (s)

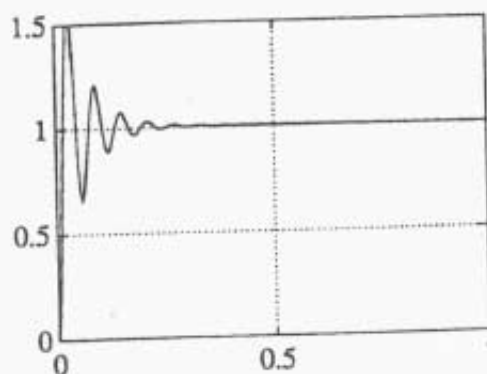
 $z_2(t)$ (cm)

(b) time (s)

Fig. 7. Step responses of sprung mass for traditional (a) and CRONE (b) suspensions:
(—) $m_2 = 150$ kg; (---) $m_2 = 225$ kg; (- · - · -) $m_2 = 300$ kg

 $z_1(t)$ (cm)

(a) time (s)

 $z_1(t)$ (cm)

(b) time (s)

Fig. 8. Step responses of wheel for traditional (a) and CRONE (b) suspensions: $m_2 = 150$ kg

6. TECHNOLOGICAL SOLUTION: THE PASSIVE CRONE SUSPENSION

The passive CRONE suspension is developed from the link between recursivity and non-integer derivation (Oustaloup, 1995). In fact, on a frequency interval, it is possible to synthesise the non-integer derivation by using N elementary spring-damper cells whose time constants are distributed recursively (Fig. 9). Each cell develops a force $f_i(t)$ defined by:

$$f_i(t) = k_i z_{r_i}(t) + b_i \frac{d}{dt} z_{r_i}(t), \quad (38)$$

in which

$$k_i = \eta^{i-1} k_1 \quad \text{and} \quad b_i = \frac{1}{\alpha^{i-1}} b_1, \quad (39)$$

α and η being the recursive factors and $z_{r_i}(t)$ the relative displacement of cell i .

From a symbolic expression of relation (38), namely

$$F_i(s) = [k_i + b_i s] Z_{r_i}(s), \quad (40)$$

the transmittance of cell i is obtained, namely:

$$\frac{F_i(s)}{Z_{r_i}(s)} = [k_i + b_i s]. \quad (41)$$

The arrangement being parallel, since

$$f_1(t) = \dots = f_i(t) = \dots = f_N(t) \quad (42)$$

and

$$\frac{d}{dt} [z_1(t) - z_2(t)] = \sum_{i=1}^N \frac{d}{dt} z_{r_i}(t), \quad (43)$$

the global suspension transmittance $C_N(s)$ is of the form

$$\frac{1}{C_N(s)} = \sum_{i=1}^N \frac{1}{1 + \frac{s}{\omega_i}}, \quad (44)$$

in which

$$\omega_i = \frac{k_i}{b_i}. \quad (45)$$

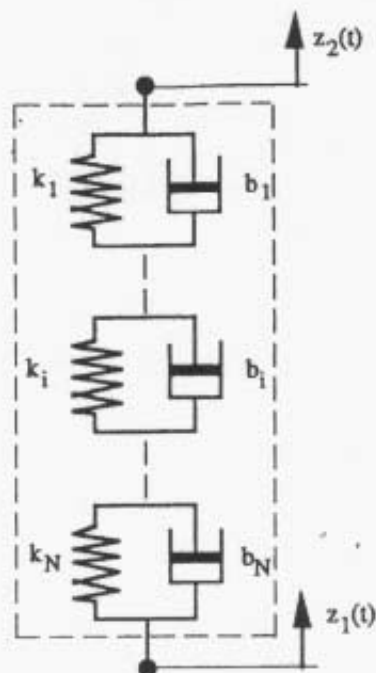


Fig. 9. Recursive arrangement of N elementary cells spring-damper

The reduction of expression (44) to the same denominator leads to the relation:

$$\frac{1}{C_N(s)} = \left(\sum_{i=1}^N \frac{1}{k_i} \right) \frac{\prod_{j=1}^{N-1} \left(1 + \frac{s}{\omega_j} \right)}{\prod_{i=1}^N \left(1 + \frac{s}{\omega_i} \right)}, \quad (46)$$

where, in the median frequency interval (Oustaloup, 1995):

$$\frac{\omega_{i+1}}{\omega_i} = \frac{\omega'_{i+1}}{\omega'_i} = \alpha \eta > 1. \quad (47)$$

Finally, the expression of transmittance $C_N(s)$ is in fact the same as relation (26), namely:

$$C_N(s) = C_0 \frac{\prod_{i=1}^N \left(1 + \frac{s}{\omega_i} \right)}{\prod_{i=1}^{N-1} \left(1 + \frac{s}{\omega_i} \right)}, \quad (48)$$

in which

$$C_0 = \frac{1}{1 + \sum_{i=1}^{N-1} \frac{1}{\eta^i}} k_1. \quad (49)$$

So, the non-integer-order suspension transmittance results from a recursive distribution of zeros and poles in the frequency interval $[\omega'_1; \omega'_N]$.

In the automotive domain, and to limit suspension dimensions, gas springs are used (Fig. 10a), each being mounted with a damper (Fig. 10b). The passive CRONE suspension (Fig. 11) is thus composed of N gas spring-damper cells in accordance with Fig. 9.

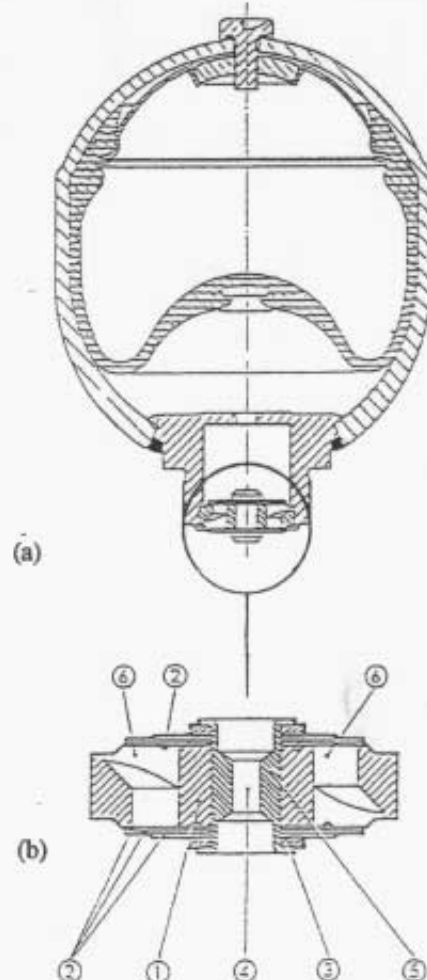


Fig. 10. Diagram of a gas spring-damper cell (a) and diagram of a damper (b): (1) body of damper; (2) valves; (3) washer; (4) hole; (5) rivet; (6) hole

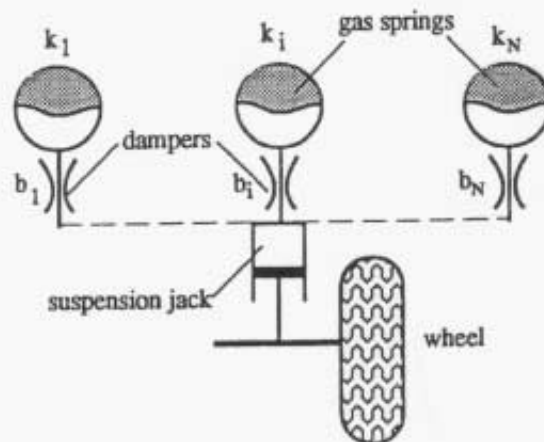


Fig. 11. Passive CRONE suspension in the automotive domain

The passive CRONE suspension is now mounted on an experimental Citroën BX. The modification to the traditional suspension is minor. This consists of a brace with three drilled and tapped holes which permit a mechanical and hydraulic bond between the suspension jack and three gas springs. Each of these

is inflated to a pressure providing a stiffness in accordance with synthesis. Each damper is mounted on a gas spring. The number of valves in each damper is determined to obtain a mean viscous friction coefficient in accordance with synthesis.

7. CONCLUSION

This paper has shown that the CRONE suspension provides remarkable performance: better robustness of stability degree versus load variations of the vehicle. Robustness is illustrated by the frequency and time responses obtained for different values of the load.

It has been shown that this robustness is due to the template which implicitly characterizes the CRONE suspension. This template characterizes the second-generation CRONE control explicitly.

From the concept of the CRONE suspension, two technological solutions have been developed (Moreau, 1995). The first, called the *passive CRONE suspension*, uses the link between recursivity and non-integer derivation. This suspension has now been mounted on an experimental Citroën BX. The second solution, called the *passive piloted CRONE suspension*, uses a continuously controlled damper. Its design permits it to be manufactured at the same cost as a traditional automobile damper. Bench tests on a prototype have validated the theoretical expectations.

REFERENCES

- Daver R., Roberti V. and Jezequel L. (1992). Nouvelle stratégie de suspension semi-active. *SIA*, n°672, pp.57-62.
- Dunwoody A. B. (1991). Digital active suspension. *SAE Transactions*, vol.100, n°6, pp.1649-1659.
- Gohring E. and Von Glasner (1993). Intelligent suspension for commercial vehicles. *Proceedings of the International Congress MV2*, Active Control in Mechanical Engineering, tome 1, pp.1-12.
- Hedrick J.K. and Butsuen T. (1990). Invariant properties of automotive suspensions. *Journal of automobile engineering*, Vol.204, n°1, pp.21-27.
- Jezequel L. and Roberti V. (1992). Behaviour of a preview semi-active suspension. In: *Mécanique-Matériaux-Electricité*, n°445, pp.32-35.
- Matlab 1992. User's guide. The Math Works Inc.
- Moreau X., Oustaloup A. and Nouillant M. (1993). La suspension CRONE: une suspension active d'ordre non entier optimal. *Proceedings of the International Congress MV2*, Active Control in Mechanical Engineering, tome 1, pp.79-92.
- Moreau X. (1995). Intérêt de la dérivation non entière en isolation vibratoire et son application dans le domaine de l'automobile. La suspension CRONE: du concept à la réalisation. *Thèse de Doctorat*, Université Bordeaux I.
- Moreau X., Oustaloup A. and Nouillant M. (1996). Comparison of LQ and CRONE methods for the design of suspension systems. *Proceedings of IFAC'96 World Congress*, San Francisco, California, USA June 30 - July 5.
- Oustaloup A. and Nouillant M. (1990). Nouveau Système de Suspension, *Brevet n°9004613*, INPI.
- Oustaloup A. (1991). *La Commande CRONE*, HERMES ED., Paris.
- Oustaloup A. (1995). *La dérivation non entière: théorie, synthèse et applications*, HERMES ED., Paris.
- Oustaloup A., Moreau X. and Nouillant M. (1993). From the second-generation CRONE control to the CRONE suspension. *Proceedings of the International Congress IEEE*, International Conference on Systems, Man and Cybernetics, Le Touquet, tome 2, pp.143-148.
- Rakheja S. and Ahmed A. K. W. (1991). Simulation of non-linear variable dampers using energy similarity. *Journal of Engineering Computation*, vol.8, n°4, pp.333-344.
- Redfield R.C. and Karnopp D.C. (1989). Performance sensitivity of an actively damped vehicle suspension to feedback variation. *Journal of dynamic Systems, Measurement, and Control*, Vol.111, n°1, pp.51-60.
- Yasuda E. and Doi S. (1991). Improvement of vehicle motion and riding-comfort by active controlled suspension system. *SAE Transactions*, vol. 100, n°6, pp.890-897.
- Yue C., Butsuen T. and Hedrick J.K. (1989). Alternative Control Laws for Automotive Active Suspensions. *Journal of dynamic Systems, Measurement, and Control*, Vol.111, n°2, pp.286-291.