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Comment on the Paper: The CRONE Suspension: Management of the Dilemma Comfort-Road Holding (by Moreau et al.)*

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Abstract. The linearisation procedure used in the modelling of the hydro-pneumatic suspension system by Moreau et al. contains an inaccuracy. Hence, the conclusion that the weight of the sprung mass is normally absent of the dynamic behaviour of the sprung mass is demonstrated to be false.

Key words: hydro-pneumatic suspension, linearisation, CRONE control

In the paper [2] of Moreau et al., an optimisation procedure of a hydro-pneumatic suspension system based on fractional order differentiation is presented. The second section, devoted to modelling of traditional hydro-pneumatic suspension of a quarter car model, contains an inaccuracy which runs into the conclusion that the weight of the sprung mass of the vehicle is absent in the equations of the dynamic behaviour of the system. As shown here-after, such a conclusion is abusive because it is admissible only in a very particular (and somewhat unrealistic) case. This comment was suggested by a detailed study of Montseny and Bidan relating specifically to some dynamical invariance questions about hydro-pneumatic suspension systems [1].

First consider the relation (15) in the paper, namely:

$$\left.\frac{\partial p_s}{\partial v_g}\right|_{\substack{p_s=p_{\text{st}}\\ v_g=v_{\text{gst}}}} = -\gamma\frac{p_{\text{st}}}{v_{\text{gst}}} = -\frac{1}{C},$$

in which C is presented as a constant: the so-called "pneumatic capacity of the sphere".

Consider also the relation (13) of the paper, namely:

$$cst = p_{\text{st}} v_{\text{gst}}^{\gamma}$$

in which, as well-known, the coefficients cst and γ depend on the particular thermodynamical situation under consideration. By combining these two equations, C is expressed as a *function* of the pressure p_{st}

$$C = \frac{cst^{1/\gamma}}{\gamma p_{\text{st}}^{(1+1/\gamma)}}.$$

On the other hand, the static pressure p_{st} (of both the oil in the suspension jack and the nitrogen in the sphere) is proportional to the sprung mass m_2, as described by the following Equation (30) of the paper (g denotes the acceleration of gravity and S_v is a constant parameter of the suspension system):

$S_v p_{\text{st}} = m_2 g,$

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which namely gives: $p_{st} = \frac{m_2 g}{S_v}$. So, because it is highly depending on the sprung mass, the parameter C cannot be considered as a characteristic constant of the suspension system in own.

Let us now study the so-called stiffness of the sphere, denoted in the paper by k_2. From the above results, it should be rewritten as:

$$k_2 = \frac{S_v^2 \gamma p_{st}^{(1+1/\gamma)}}{cst^{1/\gamma}};$$

it can therefore be expressed as the following function of m_2:

$$k_2 = \frac{S_v^{(1-1/\gamma)} \gamma (m_2 g)^{(1+1/\gamma)}}{cst^{1/\gamma}},$$

which becomes if $\gamma = 1$ (isothermal situation):

$$k_2 = \frac{(m_2 g)^2}{cst}.$$

Finally, by correcting Equations (18), (19), (20) and (21) of the paper according to the above statements, the equation (23) of the dynamic force (linearised around the equilibrium state) is actually expressed as:

$$f_{dyn}(t) = \frac{b_2}{S_v} q(t) + \frac{(m_2 g)^{(1+1/\gamma)}}{(cst\ S_v)^{1/\gamma}} \int_0^t q(\tau) d\tau.$$

or, if $\gamma = 1$:

$$f_{dyn}(t) = \frac{b_2}{S_v} q(t) + \frac{(m_2 g)^2}{cst\ S_v} \int_0^t q(\tau) d\tau.$$

So, in opposite to the conclusions of the paper of Moreau et al., the weight $m_2 g$ is significantly involved in the equations of the dynamic behaviour of the sprung mass around its static position. Note that this is not surprising due to the well-known fact that linearisation of a dynamic system can be valid only if it relates to a steady state of the *whole* dynamic state variables. In the present case indeed, the pressure of nitrogen in the sphere is such a state variable, the static equilibrium of which is essentially depending on the weight which *must* actually be present in the linearised equations. Although the above-mentioned inaccuracy has no incidence in the theoretical context of the paper because m_2 is supposed constant, it would be thoroughly different in actual situations, precisely because one of the well-known main difficulties when designing a car suspension system lies on the fact that the sprung mass m_2 is strongly variable by nature (it indeed includes the embarked load).

References

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