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From fractal robustness to non integer approach in vibration insulation : the CRONE suspension

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ABSTRACT

This contribution deals with the transposition of fractal robustness in automatics and mechanics through the CRONE control and the CRONE suspension.

Fractal robustness expresses the robustness of damping in nature, that fractality ensures through non integer derivation. This concept is, in this case, illustrated by the relaxation of water on a porous dyke, its damping being independent of the motion water mass. This robust phenomenon is paradoxical in the integer approach of mechanics, where any relaxation presents a damping linked to the carried mass.

The dynamic model which governs this phenomenon is established. It consists in a differential equation of non integer order between 1 and 2, whose solution depends on the carried water mass, but whose damping is independent of it. The transposition of this template in automatics and mechanics defines the non integer approach used by the CRONE control and the CRONE suspension.

I. INTRODUCTION : FRACTAL ROBUSTNESS AND CRONE CONTROL

I.1. Analysis of a robust natural relaxation

As early as in the 17 th century, the people who built dykes had noted the damping properties of the very disturbed dykes and particularly those forming air pockets which can be compressed by the advance of water.

The relaxation of water on such dykes has indeed been the subject of advanced experimental analyses. In particular, the time variation of the water level on the sides of the dykes has been recorded. In the case of fluvial or coastal dykes which are very damping (or absorbing) because of their porous volumic structure and rough surfacic structure, the results of these analyses reveal that :

- *the natural frequency of the relaxation is different according to whether the dyke is fluvial or coastal ;*
- *the damping of the relaxation seems to be independent of the dyke, whether it is fluvial or coastal.*

Given that the fluvial and coastal tests can be distinguished by widely differing water masses carried, the analyses seems to show that *the relaxation is characterized by a natural frequency which depends on the motion water mass and by a damping ratio which is independent of it.*

Although it appears paradoxical in the integer approach of mechanics, where any relaxation presents a damping linked to the carried mass, this result reveals the *insensitivity* of the damping ratio to a parameter at least, in this case the motion water mass. It therefore expresses the *robustness of the stability degree* of the relaxation phenomenon.

The aim of the following developments consists in determining the mathematical principle of the robustness of such a phenomenon, notably in establishing the differential equation which governs it.

I.2. Study process and water-dyke interface

Let us consider a water mass, M, the motion of which is due to its penetration in a dyke whose permeability is due to its porosity (Fig. 1). By denoting its speed by V(t), applying the fundamental law of dynamics allows to write the differential equation :

$$M\frac{dV(t)}{dt}+F(t)=0, \qquad (1)$$

in which F(t) is the reaction force of the dyke, that is to say the resultant of the forces which act on water mass M.

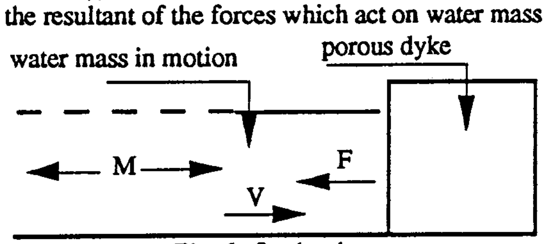

Fig. 1. Study plant

If S represents the flow section of water, it is possible to express speed V(t) versus flow Q(t), namely

$$V(t)=Q(t)\,/\,S\ ; \qquad (2)$$

moreover, force F(t) can be expressed versus dynamic pressure P(t) at the water-dyke interface, namely

$$F(t)=P(t)\,S. \qquad (3)$$

By putting expressions (2) and (3) into relation (1), one obtains a new form of the differential equation (1) :

$$\frac{M}{S^2}\,\frac{dQ(t)}{dt}+P(t)=0. \qquad (4)$$

Furthermore, by taking into account the fractality of porosity and the recursivity of fractality, we have shown [1] that *water flow Q(t) is proportional to the non integer derivative of the dynamic pressure P(t) at the water-dyke interface*, namely :

$$Q(t)=\frac{1}{\omega_0^{\,n-1}}\left(\frac{d}{dt}\right)^{n-1}P(t) \quad \text{with } 1<n<2\ ; \qquad (5)$$

this equation represents the dynamic model of the water-dyke interface.

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I.3. Non integer order differential equation as a dynamic model governing the relaxation

Putting the expression of Q(t) given by (5) into relation (4) makes it possible to establish a linear differential equation of non integer order n between 1 and 2, namely :

$$\frac{M}{S^2}\frac{1}{\omega_o^{n-1}}\left(\frac{d}{dt}\right)^n P(t) + P(t) = 0\ , \qquad (6)$$

or, under canonical form :

$$\tau^n\left(\frac{d}{dt}\right)^n P(t) + P(t) = 0\ , \qquad (7)$$

by putting

$$\tau = \left(\frac{M}{S^2}\frac{1}{\omega_o^{n-1}}\right)^{1/n}. \qquad (8)$$

I.4. Functional representation leading to a frequency template

Let us take the Laplace transform of the differential equation (7) which governs the relaxation. One obtains :

$$(\tau\ s)^n\ P(s) + P(s) = 0\ , \qquad (9)$$

from where one draws :

$$P(s) = -\left(\frac{1}{\tau\ s}\right)^n P(s)\ . \qquad (10)$$

This operational equation is translated by the functional diagram of Fig. 2 which reminds that of a free control loop (E(s) = 0). Because of an unit feedback, the direct chain determines an open loop transmittance of the form :

$$\beta(s) = \left(\frac{1}{\tau\ s}\right)^n = \left(\frac{\omega_u}{s}\right)^n, \qquad (11)$$

which is just the transmittance of a *non integer integrator* whose unit gain frequency (or transition frequency) is $\omega_u = 1/\tau$.

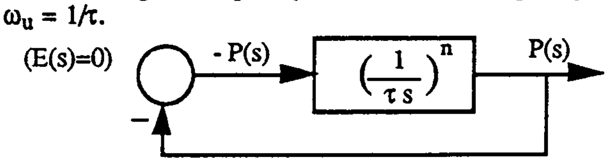

Fig. 2. Functional diagram making it possible to define an open loop transfer

Given that arg $\beta(j\omega) = n\pi/2$ with $1 < n < 2$, the Nichols locus of $\beta(j\omega)$ is a *vertical straight line* of abscissa between $-\pi/2$ and $-\pi$.

As previously, the order n-1 non integer derivation (relation (5)) is limited to a range of medium frequencies. So, the vertical straight line of abscissa $-n\ \pi/2$ is reduced to a *vertical straight line segment* lying around unit gain frequency ω_u (Fig. 3). This segment is called **open loop frequency template** (or more simply **template**).

I.5. Idea of the second generation CRONE strategy

When water mass M changes, frequency ω_u is modified in conformity with the relation

$$\omega_u = \frac{1}{\tau} = \left(\omega_o^{n-1}\frac{S^2}{M}\right)^{1/n}. \qquad (12)$$

So, the template slides on itself at the time of a variation of the water mass. Such a vertical displacement of the template ensures the constancy of phase margin Φ_m (Fig. 3) and, consequently, that of the distance to the critical point given that Φ_m then gives a significant measure of it. This expresses the robustness of stability degree. The greater the robustness, the longer the template, the displacement of the template being linked to the modification of parameter M.

In automatic control, the aim consists in obtaining a similar frequency behaviour, notably :

- *an open loop Nichols locus which forms the template so defined for the nominal parametric state of the plant ;*
- *a vertical sliding of the template at the time of a reparametration of the plant.*

The search for the synthesis of such a template defines the non integer approach that the second generation CRONE control uses [1].

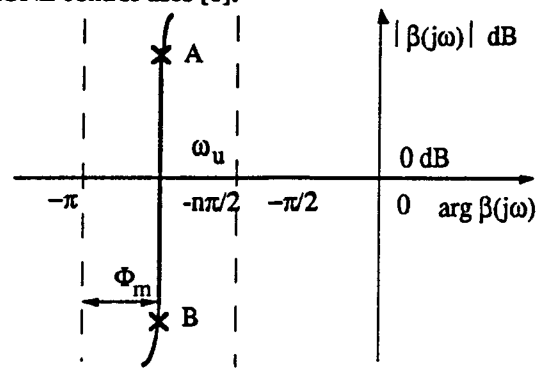

Fig. 3. A vertical straight line segment defines the template in the Nichols's plane

II. PRINCIPLE OF THE CRONE SUSPENSION

The CRONE suspension results from a traditional suspension model whose spring and damper are replaced by a mechanical system defined by a non-integer order force-displacement transmittance. This system is called the CRONE suspension because of the link with the second-generation CRONE control, i.e. the vertical template. That is why the principle of the second-generation CRONE control is used to synthesise the CRONE suspension's transmittance. The suspension parameters are determined from a constrained optimisation of a performance criterion.

II.1 Vehicle model

The basic mathematical model used for the study is composed of two mass dynamic systems consisting of the body mass m_2 (sprung mass) and the wheel mass m_1 (unsprung mass) Fig. 4. k_1 is the stiffness and b_1 the damping coefficient of the tyre. $z_0(t)$ is the deflexion of the road, $z_1(t)$ and $z_2(t)$ are the vertical displacements of the wheel and body respectively. The suspension system, located between the sprung and unsprung masses, develops a force $f_2(t)$ which can be generated by an active, semi-active or passive device [3], [4]. For example, a traditional suspension develops a force $f_2(t)$ which is a function of the relative displacement $z_{12}(t)$ and given by:

$$f_2(t) = k_2\ z_{12}(t) + b_2\ \dot{z}_{12}(t)\ , \qquad (13)$$

in which

$$z_{12}(t) = z_1(t) - z_2(t) \qquad (14)$$

and k_2 the stiffness of the spring and b_2 the damping coefficient.

The CRONE suspension develops a force $f_2(t)$ which is a function of the relative displacement $z_{12}(t)$ and which obeys symbolically to the general relation:

$$F_2(s) = C(s)\ Z_{12}(s)\ , \qquad (15)$$

in which C(s) is the suspension transmittance defined by a non integer expression.

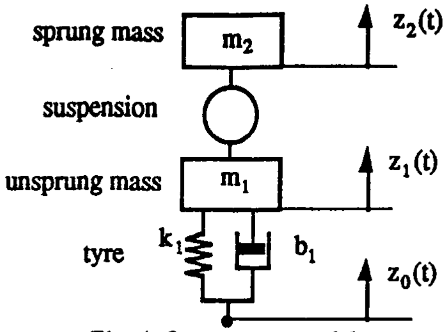

Fig. 4. Quarter car model

If it is assumed that the tyre does not leave the ground and that $z_1(t)$ and $z_2(t)$ are measured from the static equilibrium position, then the application of the fundamental law of dynamics leads to the linearised equations of motion:

$$m_1\,\ddot{z}_1(t) = f_1(t) - f_2(t) \tag{16}$$

and

$$m_2\,\ddot{z}_2(t) = f_2(t)\ , \tag{17}$$

in which

$$f_1(t) = k_1\left(z_0(t) - z_1(t)\right) + b_1\left(\dot{z}_0(t) - \dot{z}_1(t)\right)\ . \tag{18}$$

The Laplace transform of equations (16), (17) and (18), assuming zero initial conditions, are given by

$$m_1\,s^2\,Z_1(s) = k_1\,Z_{01}(s) + b_1\,s\,Z_{01}(s) - F_2(s) \tag{19}$$

and

$$m_2\,s^2\,Z_2(s) = F_2(s)\ , \tag{20}$$

in which

$$Z_{01}(s) = Z_0(s) - Z_1(s)\ . \tag{21}$$

To analyse the vibration insulation of the sprung mass, two transmittances are defined:

$$T(s) = \frac{Z_2(s)}{Z_1(s)} \quad \text{and} \quad S(s) = \frac{Z_{12}(s)}{Z_1(s)}\ . \tag{22}$$

From equations (19) and (20), the expressions of T(s) and S(s) are given by:

- for the traditional suspension

$$T(s) = \frac{k_2 + b_2\,s}{k_2 + b_2\,s + m_2\,s^2},\quad S(s) = \frac{m_2\,s^2}{k_2 + b_2\,s + m_2\,s^2}\ ; \tag{23}$$

- for the CRONE suspension

$$T(s) = \frac{C(s)}{C(s) + m_2\,s^2} \quad \text{and} \quad S(s) = \frac{m_2\,s^2}{C(s) + m_2\,s^2}\ . \tag{24}$$

To study ride comfort and road holding ability, three additional transmittances are defined:

$$H_a(s) = \frac{A_2(s)}{V_0(s)},\ H_{12}(s) = \frac{Z_{12}(s)}{V_0(s)} \text{ and } H_{01}(s) = \frac{Z_{01}(s)}{V_0(s)}\ , \tag{25}$$

in which $A_2(s)$ is acceleration of the sprung mass, $Z_{12}(s)$ suspension deflection, $Z_{01}(s)$ tyre deflection and $V_0(s)$ road input velocity. A commonly used road input model is that $v_0(t)$ is white noise whose intensity is proportional to the product of the vehicle's forward speed and a road roughness parameter [4].

II.2. Synthesis method

The synthesis method of the CRONE suspension is based on the interpretation of transmittances T(s) and S(s) which can be written as:

$$T(s) = \frac{\beta(s)}{1 + \beta(s)} \quad \text{and} \quad S(s) = \frac{1}{1 + \beta(s)}\ , \tag{26}$$

in which

$$\beta(s) = \frac{C(s)}{m_2\,s^2}\ . \tag{27}$$

The transmittances T(s) and S(s) can here be considered to be of an elementary control loop whose $\beta(s)$ is the open loop transmittance.

Given that relation (27) expresses that a variation of sprung mass is accompanied by a variation of open loop gain, the principle of the second generation CRONE control can be used by synthesising the open loop Nichols locus which traces a vertical template for the nominal sprung mass.

A way of synthesising the open-loop Nichols locus consists in determining a transfer $\beta(s)$ which successively presents Fig.5:

- an order-2 asymptotic behaviour at low frequencies to eliminate tracking error;
- an order-n asymptotic behaviour, where n is between 1 and 2, exclusively around frequency ω_u, to limit the synthesis of the non-integer derivation over a truncated frequency interval;
- an order-1 asymptotic behaviour at high frequencies, to ensure satisfactory filtering of vibrations at high frequencies.

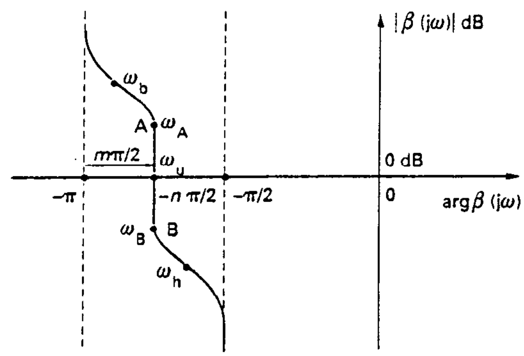

Fig. 5. Open-loop Nichols locus of the CRONE suspension

Such localised behaviour can be obtained with a transmittance of the form:

$$\beta(s) = C_0\,\frac{\left(1 + \dfrac{s}{\omega_b}\right)^m}{\left(1 + \dfrac{s}{\omega_h}\right)^{m-1}}\left(\frac{\omega_0'}{s}\right)^2 \tag{28}$$

in which:

$$\omega_b \ll \omega_A,\ \omega_B \ll \omega_h \text{ and } m = 2 - n \in\ ]0,1[\ . \tag{29}$$

Identification of equations (27) and (28) gives:

$$1/\sqrt{m_2} = \omega_0' \tag{30}$$

and

$$C(s) = C_0\,\frac{\left(1 + \dfrac{s}{\omega_b}\right)^m}{\left(1 + \dfrac{s}{\omega_h}\right)^{m-1}}\ . \tag{31}$$

The equation thus obtained defines the ideal version of the suspension. The corresponding real version [2] is defined by a transfer of integer order:

$$C_N(s) = C_0 \frac{\prod_{i=1}^{N}\left(1+\frac{s}{\omega_i'}\right)}{\prod_{i=1}^{N-1}\left(1+\frac{s}{\omega_i}\right)}, \tag{32}$$

in which:

$$\frac{\omega_{i+1}'}{\omega_i'} = \frac{\omega_{i+1}}{\omega_i} = \alpha\eta > 1;\ \frac{\omega_i}{\omega_i'} = \alpha;$$
$$\frac{\omega_{i+1}'}{\omega_i} = \eta;\ \alpha\eta = \left(\frac{\omega_h}{\omega_b}\right)^{1/N}; \tag{33}$$
$$\alpha = (\alpha\eta)^m;\ \omega_1' = \omega_b\,\eta^{1/2} \quad \text{and} \quad \omega_N = \omega_h\,\eta^{-1/2},$$

with N number of cells.

By defining the transmittances (25) with respect to $v_0(t)$, all frequencies contribute equally to their mean square values. That is why the determination of CRONE suspension parameters is based on the minimisation of a criterion J composed of the H_2-norm of the transmittances $H_a(j\omega)$, $H_{12}(j\omega)$ and $H_{01}(j\omega)$, namely:

$$J = \frac{\rho_1}{\lambda_1}\int_{\omega_b}^{\omega_h} |H_a(j\omega)|^2\, d\omega + \frac{\rho_2}{\lambda_2}\int_{\omega_b}^{\omega_h} |H_{12}(j\omega)|^2\, d\omega$$
$$+ \frac{\rho_3}{\lambda_3}\int_{\omega_b}^{\omega_h} |H_{01}(j\omega)|^2\, d\omega + \frac{\rho_4}{\lambda_4}\int_{\omega_b}^{\omega_h} |H(j\omega)|^2\, d\omega, \tag{34}$$

in which ρ_i are the weighting factors, λ_i the H_2-norm computed for the traditional suspension used for comparison, and $H(j\omega)$ the transmittance between force $F_2(j\omega)$ developed by the suspension and the road input velocity $V_0(j\omega)$, namely:

$$H(j\omega) = \frac{F_2(j\omega)}{V_0(j\omega)} = m_2\, H_a(j\omega)\ . \tag{35}$$

To obtain a significant comparison between traditional and CRONE suspension performances, a constraint is fixed for the minimal sprung mass: equal unit gain frequency of open loop $\beta(j\omega)$.

III. PERFORMANCE

The traditional suspension is rear suspension whose parameters are given by:

- sprung mass :	$150\ \text{kg} \leq m_2 \leq 300\ \text{kg}$;
- unsprung mass :	$m_1 = 28.5$ kg ;
- stiffness of tyre :	$k_1 = 155\,900$ N/m ;
- damping coefficient of tyre :	$b_1 = 50$ Ns/m ;
- stiffness :	$k_2 = 19\,960$ N/m ;
- damping coefficient :	$b_2 = 1\,861$ Ns/m ;

From this data, the constrained optimisation of the criterion J, computed with the optimisation toolbox of Matlab, provides the optimal parameters of the CRONE suspension, namely:

- for the ideal version:

$$m = 0.75\ ; \quad C_0 = 3\,174\ ; \quad \omega_b = 0.628\ \text{rd/s}\ ; \quad \omega_h = 314\ \text{rd/s}\ ; \tag{35}$$

for the real version:

$$\begin{array}{ll} N = 5\ ; & C_0 = 3\,174\ ; \\ \alpha = \omega_i/\omega_i' = 3.2067\ ; & \eta = \omega_{i+1}'/\omega_i = 1.4746\ ; \\ \omega_1' = 0.763\ \text{rd/s}\ ; & \omega_1 = 2.9\ \text{rd/s}\ ; \\ \omega_2' = 3.608\ \text{rd/s}\ ; & \omega_2 = 11.6\ \text{rd/s}\ ; \\ \omega_3' = 17.061\ \text{rd/s}\ ; & \omega_3 = 54.7\ \text{rd/s}\ ; \\ \omega_4' = 80.677\ \text{rd/s}\ ; & \omega_4 = 258.7\ \text{rd/s}\ ; \\ \omega_5' = 317.92\ \text{rd/s}\ . & \end{array} \tag{36}$$

III.1. Frequency responses

Figures 6 and 7 show frequency performances in open loop and in closed loop.

Figure 6 gives the Nichols loci $\beta(j\omega)$ for the traditional and CRONE suspensions. The phase margin varies with mass m_2 for the traditional suspension. On the other hand, phase margin is independent for the CRONE suspension, where the Nichols loci in open loop trace the template which characterizes the second generation CRONE control.

Figure 7 gives the gain diagrams of $T_2(j\omega)$ for the traditional and CRONE suspensions. For the CRONE suspension, the resonance ratio can be seen to be both weak and insensitive to variations of mass m_2. This shows a better robustness of the CRONE suspension in the frequency domain.

III.2. Step responses

Figure 8 shows the step responses of the car body for both suspensions. For the CRONE suspension it can be seen that the first overshoot remains constant, showing a better robustness for the CRONE suspension in the time domain.

IV. TECHNOLOGICAL SOLUTION

The passive CRONE suspension is developed from the link between recursivity and non-integer derivation [2]. In fact, on a frequency interval, it is possible to synthesise the non-integer derivation by using N elementary spring-damper cells whose time constants are distributed recursively (Fig. 9). Each cell develops a force $f_i(t)$ defined by:

$$f_i(t) = k_i\, z_{r_i}(t) + b_i\, \frac{d}{dt} z_{r_i}(t)\ , \tag{37}$$

in which

$$k_i = \eta^{i-1}\, k_1 \quad \text{and} \quad b_i = \frac{1}{\alpha^{i-1}}\, b_1\ , \tag{38}$$

α and η being the recursive factors and $z_{r_i}(t)$ the relative displacement of cell i.

From a symbolic expression of relation (37), namely

$$F_i(s) = \left[k_i + b_i\, s\right] Z_{r_i}(s), \tag{39}$$

the transmittance of cell i is obtained, namely:

$$\frac{F_i(s)}{Z_{r_i}(s)} = \left[k_i + b_i\, s\right]. \tag{40}$$

The arrangement being parallel, since

$$f_1(t) = ... = f_i(t) = ... = f_N(t) \tag{41}$$

and

$$\frac{d}{dt}\left[z_1(t) - z_2(t)\right] = \sum_{i=1}^{N} \frac{d}{dt} z_{r_i}(t)\ , \tag{42}$$

the global suspension transmittance $C_N(s)$ is of the form

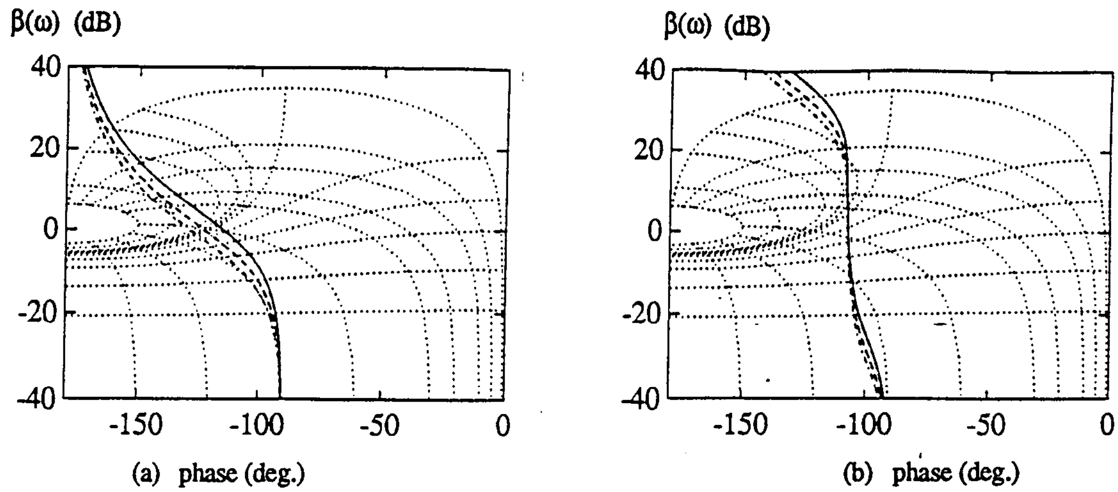

Fig. 6. Nichols loci in open loop for (a) traditional and (b) CRONE suspensions:
(————) m_2 = 150 kg; (– – – – –) m_2 = 225 kg; (– • – • –) m_2 = 300 kg

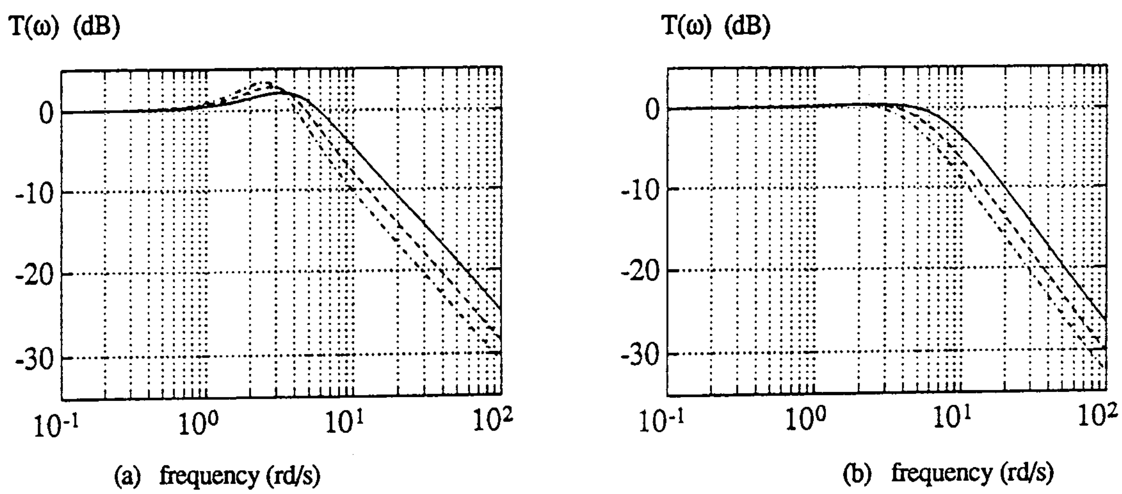

Fig. 7. Gain diagrams of T(jω) for traditional (a) and CRONE (b) suspensions:
(————) m_2 = 150 kg; (– – – – –) m_2 = 225 kg; (– • – • –) m_2 = 300 kg

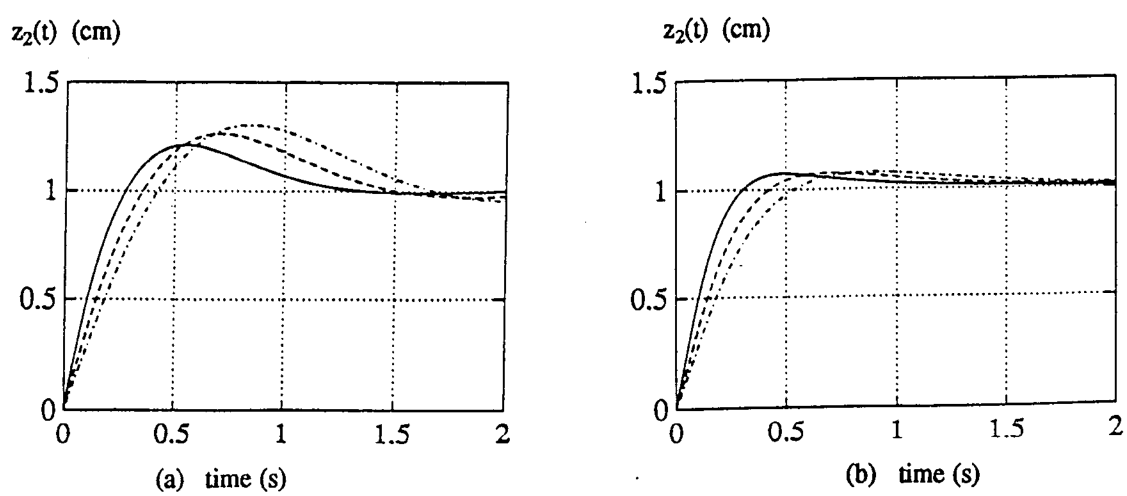

Fig. 8. Step responses of sprung mass for traditional (a) and CRONE (b) suspensions:
(————) m_2 = 150 kg; (– – – – –) m_2 = 225 kg; (– • – • –) m_2 = 300 kg

$$\frac{1}{C_N(s)} = \sum_{i=1}^{N} \frac{\frac{1}{k_i}}{1 + \frac{s}{\omega_i'}}, \qquad (43)$$

in which $$\omega_i' = \frac{k_i}{b_i}. \qquad (44)$$

The reduction of expression (43) to the same denominator leads to the relation:

$$\frac{1}{C_N(s)} = \left(\sum_{i=1}^{N} \frac{1}{k_i}\right) \frac{\prod_{i=1}^{N-1}\left(1 + \frac{s}{\omega_i}\right)}{\prod_{i=1}^{N}\left(1 + \frac{s}{\omega_i'}\right)}, \qquad (45)$$

where, in the median frequency interval [8]:

$$\frac{\omega_{i+1}}{\omega_i} = \frac{\omega_{i+1}'}{\omega_i'} = \alpha\,\eta > 1. \qquad (46)$$

Finally, the expression of transmittance $C_N(s)$ is in fact the same as relation (26), namely:

$$C_N(s) = C_0 \frac{\prod_{i=1}^{N}\left(1 + \frac{s}{\omega_i'}\right)}{\prod_{i=1}^{N-1}\left(1 + \frac{s}{\omega_i}\right)}, \qquad (47)$$

in which

$$C_0 = \frac{1}{1 + \sum_{i=1}^{N-1} \frac{1}{\eta^i}}\, k_1 . \qquad (48)$$

So, the non-integer-order suspension transmittance results from a recursive distribution of zeros and poles in the frequency interval $[\omega'_1; \omega'_N]$.

In the automotive domain, and to limit suspension dimensions, gas springs are used, each being mounted with a damper [4]. The passive CRONE suspension (Fig. 10) is thus composed of N gas spring-damper cells in accordance with Fig. 9.

The passive CRONE suspension is now mounted on an experimental Citroën BX. The modification to the traditional suspension is minor. This consists of a brace with three drilled and tapped holes which permit a mechanical and hydraulic bond between the suspension jack and three gas springs. Each of these is inflated to a pressure providing a stiffness in accordance with synthesis. Each damper is mounted on a gas spring. The number of valves in each damper is determined to obtain a mean viscous friction coefficient in accordance with synthesis.

V. CONCLUSION

The developments of this paper have shown the transposition of fractal robustness in automatics and mechanics through the CRONE control and the CRONE suspension.

The dynamic model which governs the relaxation of water on porous dyke, consists in a differential equation of non integer order between 1 and 2. Damping is indeed exclusively linked to the non integer of the differential equation imposed by the fractal dimension of the dyke. This expresses a remarkable property, that is to say fractaly determines damping in nature. The robustness of damping is illustrated by a frequency template in the Nichol's plane whose form and vertical sliding ensure the invariance of the phase margin.

The CRONE suspension which is the transposition of fractal robustness in mechanics, provides remarkable performance: better robustness of stability degree versus load variations of the vehicle. This robustness is due to the template which implicitly characterises the CRONE suspension. This template characterises the second-generation CRONE control explicitly.

From the concept of the CRONE suspension, three technological solutions have been developed. Bench tests on prototypes have validated the theoretical expectations. One of them is proposed in the case of a French car, in particular the CRONE BX Citroën which has received the "TROPHEE AFCET 95" as a national award.

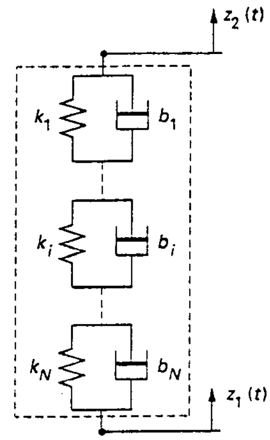

Fig. 9. Recursive arrangement of N elementary cells spring-damper

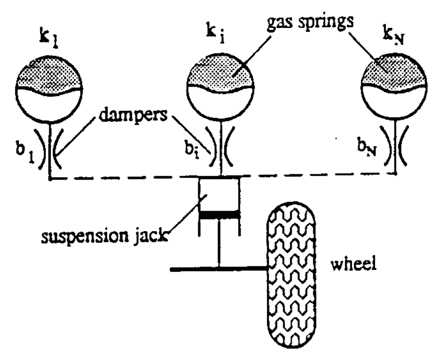

Fig. 10. Passive CRONE suspension in the automotive domain

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